A maximal covering location-based model for analyzing the vulnerability of landscapes to wildfires: Assessing the worst-case scenario

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Abstract

In this research, we study the vulnerability of landscapes to wildfires based on the impact of the worst-case scenario ignition locations. Using this scenario, we model wildfires that cause the largest damage to a landscape over a given time horizon. The landscape is modeled as a grid network, and the spread of wildfire is modeled using the minimum travel time model. To assess the impact of a wildfire in the worst-case scenario, we develop a mathematical programming model to optimally locate the ignition points so that the resulting wildfire results in the maximum damage. We compare the impacts of the worst-case wildfires (with optimally located ignition points) with the impacts of wildfires with randomly located ignition points on three landscape test cases clipped out from three national forests located in the western U.S. Our results indicate that the worst-case wildfires, on average, have more than twice the impact on landscapes than wildfires with randomly located ignition points.

Keywords: OR in natural resources; Critical infrastructure; Wildfire management; IP model; Vulnerability assessment.

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1. Introduction

Although natural fires are part of many terrestrial ecosystems [1], uncontrolled wildfires can be destructive and can cause loss of human life and property [2]. Destructive wildfires are a primary concern in places where major cities are located close to highly flammable vegetation areas, such as the Western and Southern U.S., Australia, and Mediterranean Europe [2]. There has been a sharp increase in fire events across the globe [3], and the destruction caused by wildfires appears to be worsening [4]. From 2002 through 2011, wildfires in the U.S. accounted for \$13.7 billion in total economic losses, a

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$6.9 billion increase from the previous decade[1] [5]. The deaths of 19 firefighters in 2013, the largest such loss since 1933, were part of a general trend of rising threats to lives as well as properties [5].

Wildfire risk has increased with human populations reaching further into wildlands. About 32 percent of housing units including homes, apartments and buildings in the U.S. and 10 percent of all lands with houses are situated in the wildland-urban interface (WUI; the zone of transition between natural land and human development) [6], and WUI is expected to continue to grow [7]. Homes located in the WUI have a high probability of exposure to wildfire, regardless of vegetation type or potential fire size [8]. Along with increasing wildfire risk, the costs associated with wildfire management are increasing. The United States Department of Agriculture (USDA) reported that more than $1.6 billion is spent annually by state forestry agencies on wildfire protection, prevention, and suppression [8]. To reduce the consequences of catastrophic wildfires, planning effective mitigation programs is essential.

Risk assessment has increasingly become a key input to wildfire prevention and mitigation decision making processes [9, 10, 11, 12, 13]. Miller and Ager have reviewed the recent advances in risk analysis for wildfires management [14]. Determining the vulnerability of a system is an important component of risk assessment, which is employed to help develop risk mitigation strategies to counter risks [15]. Vulnerability assessment studies identify weak points in the system, and focus on defined threats that could compromise the system's ability to meet its intended function. To our knowledge, no risk assessment study has considered the worst-case wildfires, and there has not been any pilot risk assessment for a potential arson-induced wildfire that utilizes coordinated multiple ignition points. The results of such a study can be used in strategic planning efforts for risk mitigation against a threat, especially when available resources and funds are limited. This paper aims to fill this gap by proposing a mathematical programming model to study the vulnerability of landscapes to wildfires in the worst-case scenario.

Operations Research (OR) specialists have worked with fire managers to develop decision support systems that can help improve fire management; however, there remain substantial gaps between wildfire managers' needs and the decision support systems used [16]. Linear programming and mixed integer programming (MIP) have been frequently used in wildfire management (e.g., [17, 18, 19, 20, 21, 22]). Other approaches such as heuristics [23, 24, 25, 26, 27], nonlinear programming [28], goal programming [29], stochastic programming [30, 31, 32], stochastic dynamic programming [33, 34], and robust optimization [35, 36] also have been used in wildfire management. There have also been some simulation-optimization applications in wildfire research (e.g. [37]). Interested readers can find some review papers regarding the applications of OR in wildfire

[1] These values are not adjusted for inflation.

management; e.g. [38, 39, 40]. In this research, we develop a mathematical programming model to evaluate the maximum impact of a wildfire on a landscape. We use the model to analyze the vulnerability of landscapes to wildfires based on the impact of the worst-case scenario ignition locations.

Although wildfires can start from anywhere on a landscape, the location and number of ignition points can be an important factor that impact the resulting wildfire spread. Using our developed optimization model, we investigate the effect of ignition locations on wildfires and identify the potential ignition locations which result in a wildfire with the maximum impact on a landscape. To model wildfire's behavior on a landscape, we use FlamMap [41], a fire behavior mapping and analysis program. We consider wildfires that contain a single and multiple ignition points, such as wildfires caused by lightning [42]. The proposed model is then used to evaluate the impact of wildfire on three landscape test problems clipped out of three national forests in the Western U.S.

We believe this to be the first study that analyzes the worst-case vulnerability of landscapes to wildfires with regard to the location of ignition sites. Our ultimate goal in this paper is to evaluate the impact of the worst-case wildfires and to assess the vulnerability of landscapes to these wildfires. Identifying the highly vulnerable areas of landscapes can help wildfire managers in wildfire risk mitigation planning such as fuels treatment scheduling and fire suppression preparedness planning.

The remainder of the paper is organized as follows: Fire modeling details and the proposed mathematical model are presented and explained in section 2. In section 3, the model's functionality is tested on three landscape test problems, and the results are presented. Finally, section 4 discusses the results and implications of our research.

2. Problem description and model formulation

2.1. Problem description

Our objective is to identify ignition locations of a wildfire that pose the maximum damage to the landscape. Damage or impact (used interchangeably through this paper) can be evaluated as the percentage of the landscape burned, or the value lost to fire. For the latter, the value of vegetation type, e.g. commercial timber, and the value of wildland-urban interface (WUI), if any, are used. We consider a landscape divided into a number of raster cells, and use FlamMap to model fire spread characteristics in each cell. If X is the set of vector x indicating the cell(s) from which a fire originates, and $f(x)$ is a function representing the corresponding impact of the fire on the landscape, then the research problem can be defined as identifying the ignition points, represented by vector x, of a fire that has the largest impact on the landscape, or equivalently to find x for which $f(x)$ is the maximum. We formulate the problem as a network optimization problem and later in section 3 test it on three landscape test cases.

The primary assumptions for the research problem studied in this paper are as follows:

i. the ignition points of wildfires are randomly distributed across the landscape;

ii. multiple fires can start at any location in the landscape; however, for simplicity, we assume that the physical interaction of fires is negligible, and therefore fire behavior and characteristics do not change in presence of another fire;

iii. if multiple fires are ignited, they are all ignited at the same time and burn for the same duration and under the same fire weather conditions;

iv. the areas outside the boundaries are unburnable;

v. when wildfire reaches the center of a cell, that cell is assumed burned; and

vi. fire spreads in an elliptical shape within each cell.

2.2. Modeling the spread of wildfire

To model the spread of wildfire as a network optimization problem, we represent a landscape with a raster map divided into grid cells. If we represent the center of each cell as a node, and connect neighboring cells with directed arcs, then the landscape can be represented with a directed network (Fig.1). As shown in Fig.1 we use bidirectional arcs for modeling the spread of fire, implying that fire can burn up and down slopes and with and into the wind. To model the spread of fire in the landscape, we use the minimum travel time algorithm (MTT) [43] to analyze a scenario where multiple wildfires start at the same time across a landscape.

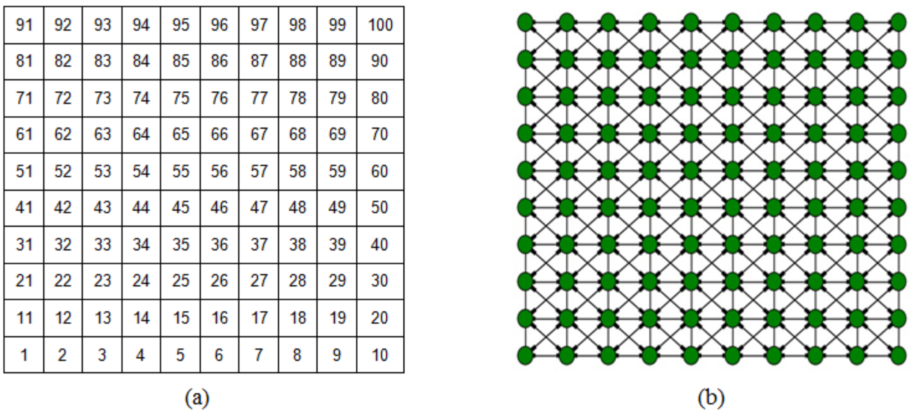

Fig. 1: (a) A landscape modeled as a 10 by 10 raster cells, (b) The directed-network representation of the landscape used to optimize wildfire spread

We use FlamMap to calculate the Rate of Spread (ROS) along with the major fire spread direction in each cell. The major fire spread direction in each cell represents the direction in that cell in which fires spread with the fastest speed. Fires can also spread along other directions, but at slower speed [44]. We use formulas (1) and (2) to calculate ROS along other directions.

$$ROS = \frac{b^2-c^2}{b-c\times COS(\theta)} \qquad 0 \leq \theta < \frac{\pi}{2} \tag{1}$$

$$ROS = \frac{b^2-c^2}{b+c\times COS(\pi-\theta)} \quad \frac{\pi}{2} \leq \theta < \pi \tag{2}$$

θ is the angle between major fire spread direction in each cell computed by FlamMap and the fire spread direction from this cell to the center of adjacent cells. In this formula b and c are outputs of FlamMap and are standard parameters used to describe the ellipse of fire spread. For more information we refer the reader to [45].

2.3. Mathematical formulation

The model uses the following notation:

Sets and Indices

- d is the expected fire duration (minutes);
- C is the set of raster cells in a landscape indexed with r, i and j ;
- N_i is the set of raster cells adjacent to cell i ;

Parameters

- $F_{i,j}$ is the distance (meters) from the center of cell i to the center of adjacent cell j ;
- $R_{i,j}$ is the rate of fire spread (meters per minute) from cell i to adjacent cell j (computed using equations (1) and (2)) ;
- $t_{i,j}$ is the fire spread time (minutes) from cell i to adjacent cell j , $t_{i,j} = \frac{F_{i,j}}{R_{i,j}}$;
- B is the number of ignition points;
- V_r is the value of cell r lost to the fire;
- $L_{j,r}$ is the length of the shortest path (or equivalently the minimum travel time) from cell j to cell r, $L_{j,r}$ is the sum of fire travel time on all the links of a shortest path that starts from cell j and ends at cell r;
- $H_{j,r}$ is 1 if $L_{j,r} \leq d$, and 0 otherwise ($H_{j,r}$ implies whether cell r is reached by a wildfire that starts at cell j within duration d);

Variables

- z_j 1 if a wildfire starts at cell j, 0 otherwise;
- y_r 1 if cell r is reached by a wildfire within duration d, 0 otherwise.

In wildfires, it is not only how much of the landscape that is burned and damaged that matters, but also monetary losses. Therefore, the objective function of the model should compute the total damage including a monetary value lost to wildfire. The model identifies the optimal locations of ignition points such that the resulting wildfire has the maximum impact on the landscape. The optimization model is specified as follows:

$$[\boldsymbol{MCWVA}] \;\; \max f = \textstyle\sum_{r \in C} V_r \times y_r \tag{3}$$

$$y_r \leq \textstyle\sum_{j \in C} H_{j,r} \times z_j \qquad \forall r \in C \tag{4}$$

$$\textstyle\sum_{j \in C} z_j \leq B \tag{5}$$

$$y_r \in \{0,\, 1\} \qquad \forall r \in C \tag{6}$$

$$z_j \in \{0,\, 1\} \qquad \forall j \in C \tag{7}$$

This model is based on the maximal covering location problem [46]. We term the model "maximal covering location-based wildfire vulnerability assessment," or MCWVA. The MCWVA finds the set of fire ignition points that can *cover* the maximal amount of landscape value. The *coverage* of a particular ignition point is the accumulative values of landscape raster cells that are burned within time d. ~~In this model, the shortest paths, their lengths ($L_{j,r}$), and accordingly $H_{j,r}$ parameters are pre-computed and then entered into the model as input parameters.~~

The objective function (3) maximizes the total ~~loss of~~ values of the cells in the landscape exposed ~~due~~ to wildfires. Constraints (4) are the burn constraints, and set the values of the binary variable y_r.The variable y_r can only be 1 if a fire is ignited at one or more ignition points that can reach r. Constraint (5) controls the number of ignition points. Constraints (6) - (7) restrict the variables to binary values. The model can consider unburnable cells or treated cells (e.g. cells with fuel breaks) if such data are available. For example, if cell i is a treated cell then this affects the fire spread time $t_{i,j}$ from cell i to any adjacent cell j. We can increase $t_{i,j}$ by a constant greater than d so that it lengthens the paths that go through cell i, and, therefore, prohibits wildfires from spreading through cell i. One can also define the ignition probability for each cell in the landscape such that for unburnable cells or treated cells, the corresponding ignition probability is zero. There might be parts of the landscape that have more fire incidences, so those cells should have higher ignition probabilities. For this reason, historical wildfire records can be used to estimate the average annual wildfire occurrence rates in each cell [47].

2.3.1. Model Data

In our model a landscape is modeled as a raster grid, which is represented by a network (as shown in 1). Before running the MCWVA model, FlamMap must be run for the landscape in order to compute the major fire spread direction (θ) and b and c (standard parameters used to describe the ellipse of fire spread). These parameters are used in equations (1) and (2) to compute the rate of fire spread $R_{i,j}$ for any potential ignition point i to any of its adjacent point j in the landscape. Denote $F_{i,j}$ as the distance (in meters) from point i to point j. Then the fire spread time from point i to point j can

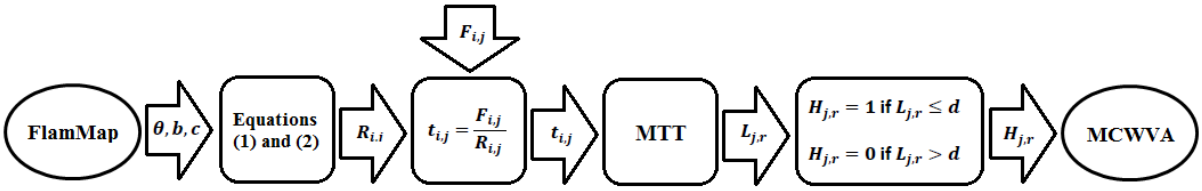

Figure 2: The process of modeling fire behavior using FlamMap and the MCWVA model

be calculated using $t_{i,j} = \frac{F_{i,j}}{R_{i,j}}$. For each potential ignition point j and an arbitrary point r in the landscape, we use the minimum travel time algorithm (MTT) [43] to compute the minimum travel time path from j to r. Let $L_{j,r}$ be the minimum travel time for the fire to reach point r from the ignition point j. Now we can say that if $L_{j,r} \leq d$ (fire can travel from j to r within duration d using the path with the minimum travel time), then cell r can be reached and burned by the fire that is ignited in cell j. That is, if $L_{j,r} \leq d$, we set the parameter $H_{j,r}$ equal to 1, otherwise 0. Thus, it is the parameter $H_{j,r}$ in MCWVA that are computed by FlamMap (see 2).

In the next section, we use MCWVA to investigate the impact of wildfires with optimally located ignition points. We also compute the average impact of wildfires over all possible ignition location scenarios. The current model can be extended to compute the expected loss due to wildfires across a possible fire duration distribution [48], instead of a fixed fire duration. Given the probability for each fire duration, it can be added to the objective function.

3. Model demonstration

In this section, we use the MCWVA model to assess the impacts of the worst-case wildfires on three landscape test problems located in the western U.S., where large wildfires are common. For these landscapes, we compare two scenarios: the worst-case wildfires with optimally located ignition points and wildfires with randomly located ignition points. For the former, we use our MCWVA model to compute the maximum impact of wildfires based on their ignition locations, and for the latter we compute the average impact of wildfires with ignition points randomly located across the landscape. For this reason, we conduct a series of experiments to consider the impact of wildfires on different landscapes, with different fire durations, and different wind speed scenarios. We also run a series of experiments to compute the impact of wildfires in presence of WUI in a landscape. These experiments are discussed in details in the following sections.

We used the LANDFIRE database to obtain landscape files (LCP) for the landscapes under study. LANDFIRE data are commonly used in wildland fire simulation modeling, as they are standardized, and updated regularly to adjust to disturbances such as wildfires, fuels treatment, and urban development [11]. Landscape files (LCP) contain spatial data themes such as fuel models, elevation, slope, aspect, and canopy characteristics. We use these data as inputs of FlamMap to model fire behavior and spread in each cell of the landscapes. FlamMap inputs these data, along with wind speed, wind direction, and fuel moisture conditions to compute rate of spread and the major fire spread direction in each cell. We use the outputs of FlamMap (the rate of spread and the major fire spread direction in each cell) to model fire spread in the landscapes using minimum travel time algorithm. The details of the landscape cases are discussed in the following section.

3.1. Landscape test problems

The first case is the 6307 km^2 Santa Fe National Forest in northern New Mexico. A prevailing west to east wind with 12 miles per hour (19.31 km per hour) speed is assumed for this case. The second case is the 3979 km^2 Umpqua National Forest at the western slopes of Cascade Mountains in Oregon. The same wind condition is assumed.

Fig. 3: The approximate locations of the forest landscape cases in the U.S. to model the evaluate of the worst-case wildfires (retrieved from [49])

The third case is the 3334 km^2 San Bernardino National Forest located in the San Bernardino Mountains in southern California. For this case a prevailing west to east wind with 12 miles per hour speed is again assumed (we also study this case under slower and faster wind speed conditions). Fig. 3 shows the approximate locations of these case study landscapes.

Although modeling these cases into rasterized networks with high number of cells makes the model more accurate, as the size of the networks increases, the model becomes more difficult to solve [44]. We clip an area of 3 km by 3 km from the first and second landscapes. To test the capability of the model for a larger landscape, we clip an area of 4.2 km by 4.2 km from the third landscape and rasterize them into networks with 25 by 25 (625) square cells, each 120 m by 120 m wide, for the first two landscapes, and 35 by 35 (1225) square cells, each 120 m by 120 m wide, for the third landscape. To quantify fire behavior on these landscapes, we use FlamMap 5.0 to calculate the rate of spread and fire spread directions.

Table 1: Initial fuel moisture conditions used in FlamMap to model the worst-case wildfires in the landscape cases

1 hour fuel moisture	6
10 hour fuel moisture	7
100 hour fuel moisture	8
Herbaceous fuel moisture	60
Live woody fuel moisture	90

We use the same initial fuel moisture conditions for all three landscape test problems in our study (Table 1). FlamMap uses Geographic Information Systems (GIS) data, landscape characteristics, fuel moisture, and wind conditions and outputs rate of spread and major fire spread directions, the fire behavior characteristics for each cell which are used in modeling fire behavior in a landscape.

3.2. Computational results

In this section, we run a set of experiments to find the effect of the locations of ignition points on the damage that wildfires can cause. Therefore, we compare two scenarios: (1) wildfires with random ignition points ("random wildfires"), and (2) wildfires with optimally located ignition points ("worst-case wildfires"). In worst-case wildfires, the ignition locations are selected optimally through solving MCWVA model. Fig. 4 shows the fire foot print after 24 hours for a sample random wildfire and the worst-case wildfire with one ignition point for the Santa Fe landscape. The worst-case wildfire with an optimally located ignition point has much larger impact on the landscape than the sample random wildfire (see Fig. 4).

To compare these wildfires, we conduct a series of experiments by which we also test the effect of the number of ignition points, fire duration, and wind speed. In the first set of experiments we assume cells have the same value across all the landscapes. We compute the impact of wildfires as percentages of landscapes burned. Through these experiments, we can see the impact of wildfires on different landscapes as well.

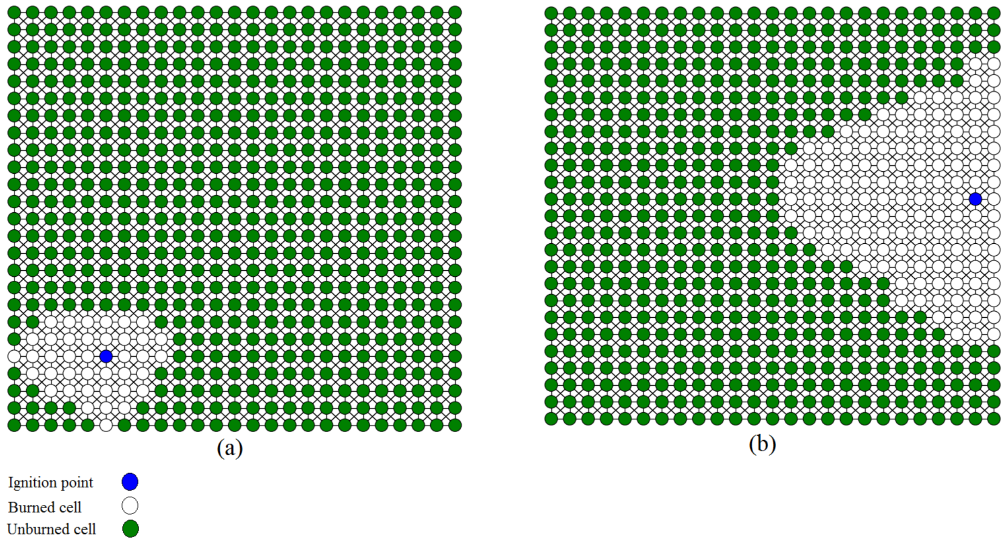

Fig. 4: Fire footprint after 24 hours for the Santa Fe National Forest landscape for (a) a sample random wildfire with single ignition point, (b) the worst-case wildfire with single ignition point.

In the second set of experiments, we test the effect of wind speed on wildfires' impact. In the last set of experiments, we assume that part of the landscape is occupied by WUI, and, therefore, not all cells have equal value. In this experiment, we test the impact of worst-case wildfires in presence of WUI.

To calculate the impact of wildfires with optimally located ignition points, we solve the MCWVA model for the three landscape cases. We implement the model formulation using Python 2.7 and solve it with Gurobi 6.0 [50]. All tests are performed on a computer with Intel Core i5 2520M processor at 2.5 GHz and 8 GB RAM. By solving the model to optimality, it gives us the optimal location(s) of ignition point(s) for a wildfire with the maximum damage it can cause.

In all of the following experiments, we compare the two wildfire cases (random wildfires and worst-case wildfires) for different number of ignition point scenarios, by systematically increasing the number of ignition points from one to five. To calculate the impact of random wildfires, in which the ignition points are randomly located, we compute the average impact of wildfires, for all scenarios of ignition locations, for one and two ignition points. However, for three and more ignition points, computing the average impact of wildfires requires tremendous computational effort. For example, for a three ignition point scenario, we would need to compute the average impact of wildfires for C_3^{625} scenarios (number of 3-combination from a set with 625 elements), which entails more than 40 million scenarios for the first two landscapes, and more than 300 million scenarios for the third landscape (C_3^{1225}). Therefore, we use Monte Carlo simulation for 3, 4, and 5 ignition point scenarios. We take a random sample of 5,000 possible ignition location scenarios, and after finding the average and standard deviation of the impact of wildfires for each case, we build 95% confidence intervals for comparison. The experiments are described in the following sections.

3.2.1. The impact of wildfires on different landscapes

In this section, we run a set of experiments on the three landscape test problems to investigate the impact of two cases of wildfires, random wildfires, and worst-case wildfires. We compute the impacts of these wildfires under three fire duration scenarios, 12, 18 and 24 hours. We assume the same rate of spread for fire for all these scenarios, though in real world, fire spread may vary diurnally. For random wildfires, we compute the average impact, and the 95% confidence intervals for 5,000 randomly selected Monte Carlo samples. We assume that all cells are homogeneous and have equal values ($V_r = 1 \ \forall r \in C$). Thus, the impacts of wildfires can be presented as the percentages of the landscape burned. Table 2 shows the percentages of each landscape burned by worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)).

Table 2: The percentages of study landscapes burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration scenarios.

Fire Duration	Landscape Name	WCWF (1)	RWF (1)	WCWF (2)	RWF (2)	WCWF (3)	RWF (3)	WCWF (4)	RWF (4)	WCWF (5)	RWF (5)
12 hours	Santa Fe	7.84	2.72	14.72	5.28	20.96	8.00	25.92	10.56	30.56	12.96
	Umpqua	8.96	2.40	16.64	4.80	22.24	7.20	27.20	9.44	32.16	11.68
	San Bernardino	11.84	4.73	20.24	9.31	28.24	13.63	36.00	17.63	42.69	21.47
18 hours	Santa Fe	13.92	5.12	23.84	11.36	33.28	16.48	42.40	21.28	49.44	25.76
	Umpqua	17.60	5.28	28.16	10.24	36.48	15.04	44.32	19.36	52.00	23.52
	San Bernardino	20.73	10.20	36.33	19.27	49.71	27.27	61.63	34.45	72.90	40.82
24 hours	Santa Fe	21.12	9.92	35.84	18.88	48.32	26.88	59.20	33.92	68.80	40.00
	Umpqua	25.60	9.28	38.56	17.44	50.08	24.80	60.64	31.36	69.76	36.80
	San Bernardino	31.84	17.06	55.43	30.94	76.49	42.12	87.02	51.51	93.71	59.02
Average		17.72	7.41	29.97	14.20	40.64	20.16	49.37	25.50	56.89	30.23

Because of the limited space in Table 2, we only show the average percentages of landscapes burned for random wildfire (RWF) cases, and not include the confidence intervals (to see the confidence intervals refer to Table 6 in the Appendix). For the three landscape cases, the differences between the average impacts of random wildfires (based on the number of ignition points) are statistically significant at 95% significance level (none of the computed confidence intervals overlap, see Table 6 in the Appendix). With the same number of ignition points and under the same fire duration scenario, the worst-case wildfires and random wildfires have different impacts on landscape cases (the differences are statistically significant at 95% significance level).

For wildfires with the same number of ignition points, the worst-case wildfires cause more than twice the damage than random wildfires (Table 3).

This difference is marked for wildfires with only one ignition point; the WCWF(1) causes approximately three times on average more damage to the landscapes than RWF(1), when the wildfire lasts for 12 hours (the average of 2.88, 3.73 and 2.50 is about 3.04, see

Table 3: The ratios of percentages of landscapes burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration scenarios

Fire Duration	Landscape Name	$\frac{WCWF(1)}{RWF(1)}$	$\frac{WCWF(2)}{RWF(2)}$	$\frac{WCWF(3)}{RWF(3)}$	$\frac{WCWF(4)}{RWF(4)}$	$\frac{WCWF(5)}{RWF(5)}$	Average
	Santa Fe	2.88	2.79	2.62	2.45	2.36	2.62
12 hours	Umpqua	3.73	3.47	3.09	2.88	2.75	3.18
	San Bernardino	2.50	2.17	2.07	2.04	1.99	2.16
	Santa Fe	2.72	2.10	2.02	1.99	1.92	2.15
18 hours	Umpqua	3.33	2.75	2.43	2.29	2.21	2.60
	San Bernardino	2.03	1.89	1.82	1.79	1.79	1.86
	Santa Fe	2.13	1.90	1.80	1.75	1.72	1.86
24 hours	Umpqua	2.76	2.21	2.02	1.93	1.90	2.16
	San Bernardino	1.87	1.79	1.82	1.69	1.59	1.75
	Average	2.66	2.34	2.19	2.09	2.02	2.26

Table 3). When the number of ignition points increases, the difference between the two wildfire cases gradually decreases (Table 3). The worst-case wildfires over random wildfires ratio goes from 2.66 for wildfires with one ignition point to 2.02 for wildfires with five ignition points.

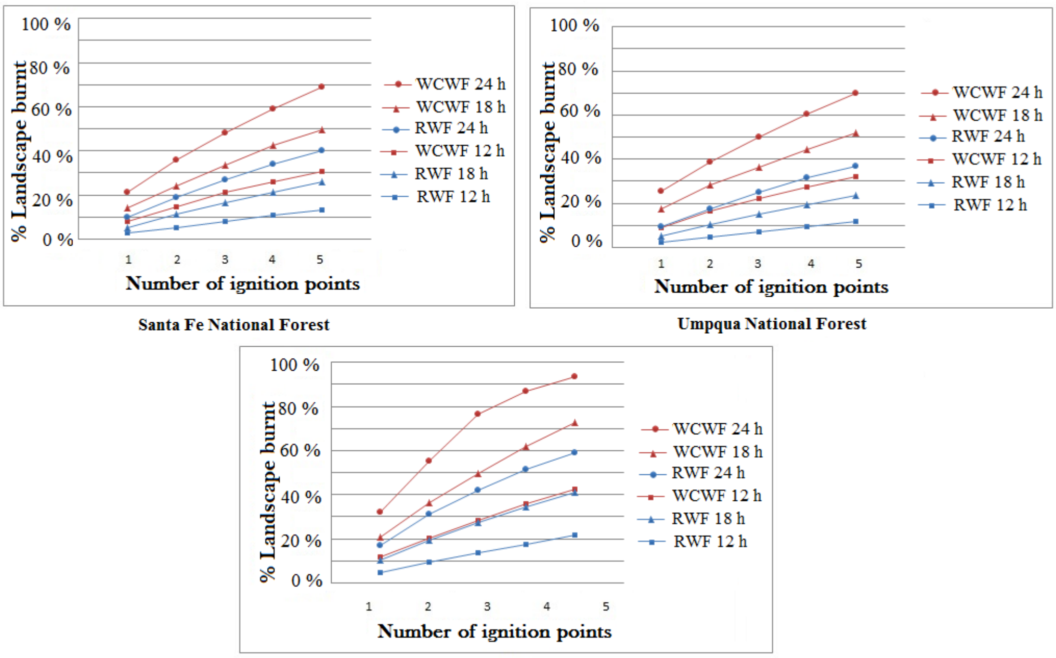

Fig. 5: The percentage of the three landscape test problems burned with random wildfires (represented by RWF) and the worst-case wildfires (represented by WCWF) under different number of ignition points, and different fire duration scenario.

As the results show, the worst-case wildfires have higher impacts on the landscapes than random wildfires (Fig. 5). In addition, wildfires have different impacts on different landscapes. The worst-case wildfires and random wildfires both have higher impact on the San Bernardino test landscape than the other two test landscapes (Fig. 5). Also, the difference between the impact of the worst-case wildfires and the average impact of random wildfires is greater for the Umpqua test landscape than the San Bernardino test landscape (Table 3). These differences are likely due to landscape characteristics which impact the rate of spread and major fire spread direction. The worst-case wildfires and random wildfires both cause more damage on landscapes when fires last longer; however, the worst-case wildfires on average spread faster and cause more damage over shorter times than random wildfires cause over longer times (Fig. 5). For example, the impact of the worst-case wildfires over 12 hours and 18 hours are respectively more than the impact of random wildfires over 18 hours and 24 hours.

3.2.2. The impact of wildfires under different wind speed conditions

In addition to landscape characteristics, wind speed also has a major impact on fire behavior [51]. In the previous set of experiments, we assumed the same wind speed conditions for all three landscape test problems. In this section, we test the impact of wildfires under three different wind speed scenarios. By doing so, we can obtain a more robust conclusion about the effect of ignition locations on the impact of wildfires on landscapes. For this reason, we run a set of experiments on the San Bernardino test landscape (the largest test landscape with 35 by 35 cells) to investigate the impact of wildfires under three different wind speed scenarios: 8, 12 and 16 *mph* (12.87, 19.31, 25.75 *kph* respectively). As we discussed before, of the three cases, the San Bernardino case has the least difference between worst-case wildfires and random wildfires (we pick the weakest case for this experiment). The results show that for higher speed winds, wildfires cause more damage; the higher the wind speed, the more damage the wildfires cause (Table 4 and Fig. 6). In this experiment, under different wind speed scenarios, the worst-case wildfires still have a greater impact on the landscape than random wildfires (Table 4 and 5; for 95% confidence intervals for random wildfires see Table 7 in the appendix).

Table 4: The percentages of the San Bernardino landscape burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration and wind speed scenarios.

Fire Duration	Wind (MPH)	WCWF (1)	RWF (1)	WCWF (2)	RWF (2)	WCWF (3)	RWF (3)	WCWF (4)	RWF (4)	WCWF (5)	RWF (5)
	8	11.67	3.92	20.08	7.67	26.29	11.18	32.00	14.61	37.47	17.88
12 hours	12	11.84	4.73	20.24	9.31	28.24	13.63	36.00	17.63	42.69	21.47
	16	13.06	5.88	23.67	11.51	33.71	16.65	43.18	21.47	50.53	26.04
	8	20.24	8.33	32.73	15.84	43.27	27.61	53.71	28.73	62.53	34.29
18 hours	12	20.73	10.20	36.33	19.27	49.71	27.27	61.63	34.45	72.90	40.82
	16	21.63	12.57	42.29	23.43	60.82	32.73	74.04	40.90	84.49	47.92
	8	31.18	13.96	48.24	25.63	62.37	35.43	72.65	43.59	83.67	50.69
24 hours	12	31.84	17.06	55.43	30.94	76.49	87.02	42.12	51.51	93.71	59.02
	16	35.10	20.82	62.61	36.90	84.33	49.39	94.12	59.27	96.65	66.86
Average		21.92	10.83	37.96	20.06	51.69	33.43	56.61	34.68	69.40	40.55

For wildfires with the same number of ignition points, and for the same fire duration scenario, the worst-case wildfires under low wind speed condition have higher impact on the landscape than random wildfires under higher wind speed condition (Fig. 6). For example, the worst-case wildfires with the 8 *mph* wind condition have higher impact on the landscape than random wildfires with the 16 *mph* wind condition. For wildfires with one and two ignition points, the impact of worst-case wildfires is on average twice the impact of random wildfires (Table 5). This difference decreases as the number of ignition points and the fire duration increase.

Table 5: The ratios of percentages of the San Bernardino landscape burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration and wind speed scenarios.

Fire Duration	Wind (MPH)	$\frac{WCWF(1)}{RWF(1)}$	$\frac{WCWF(2)}{RWF(2)}$	$\frac{WCWF(3)}{RWF(3)}$	$\frac{WCWF(4)}{RWF(4)}$	$\frac{WCWF(5)}{RWF(5)}$	Average
	8	2.98	2.62	2.35	2.19	2.10	2.45
12 hours	12	2.50	2.17	2.07	2.04	1.99	2.16
	16	2.22	2.06	2.02	2.01	1.94	2.05
	8	2.43	2.07	1.91	1.87	1.82	2.02
18 hours	12	2.03	1.89	1.82	1.79	1.79	1.86
	16	1.72	1.80	1.86	1.81	1.76	1.79
	8	2.23	1.88	1.76	1.67	1.65	1.84
24 hours	12	1.87	1.79	1.82	1.69	1.59	1.75
	16	1.69	1.70	1.71	1.59	1.45	1.62
Average		2.19	2.00	1.93	1.85	1.79	1.95

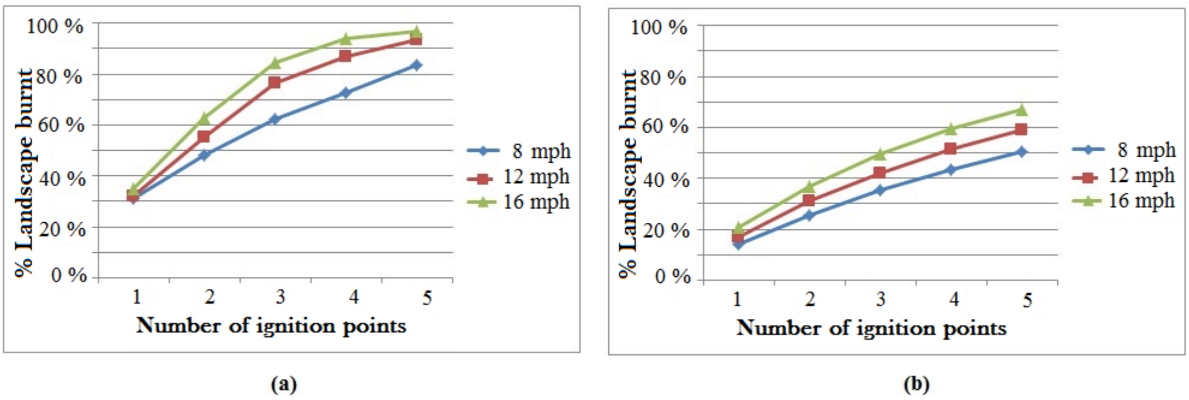

Fig. 6: The percentages of the San Bernardino landscape case burned with: (a) the worst-case wildfires, and (b) random wildfires; for different number of ignition points when wildfires last for 24 hours.

3.2.3. The impact of wildfires in presence of Wildland-Urban Interface

To investigate the impact of wildfires on landscapes in the presence of WUI, we run another set of experiments on San Bernardino landscape (the largest landscape with 35 by 35 cells). In this set of experiments, we assume that about ten percent of the landscape contains intermix WUI. In intermix WUI, as opposed to interface WUI, houses mingle with wildland fuels [8], allowing the cells containing WUI to be ignitable points. To address WUI losses due to wildfires, we include the value of each cell in the model. By doing so, we can also address cases where cells have different values depending on the vegetation type. In this experiment, WUI locations are distributed arbitrarily through the landscape. To set a value for each cell in the corresponding network, we assume a non-WUI cell has a value of 0.4, the same value that Wei [44] uses for non-commercial timber forest. As it is difficult to estimate the damage to a WUI cell, including damage to human life and property, we follow Wei [44] and use a value of 1.4 for cells containing WUI (and non-commercial timber). These values are unit-less. However, the RAVAR [52] resource evaluation method along with the real locations of WUI and vegetation types can be used to assign a value to each cell. We assume that all wildfires burn for 24 hours. The objective of the mathematical optimization model is to locate the ignition points of a wildfire that causes the maximum damage. Therefore, we expect the model to locate the ignition points adjacent to cells with higher values (WUI cells), and thus the resulting worst-case wildfire causes more damage to WUI cells than random wildfires causes. Fig. 7(a) shows the value lost due to wildfires that last for 24 hours considering different numbers of ignition points, and Fig. 7(b) shows the percentage of WUI cells that are burned by the two types of wildfires, the worst-case wildfires and random wildfires. As expected, the worst-case wildfires still have higher impact on the landscape and pose more risk (more than two times on average) to WUI than random wildfires (Fig. 7(b)).

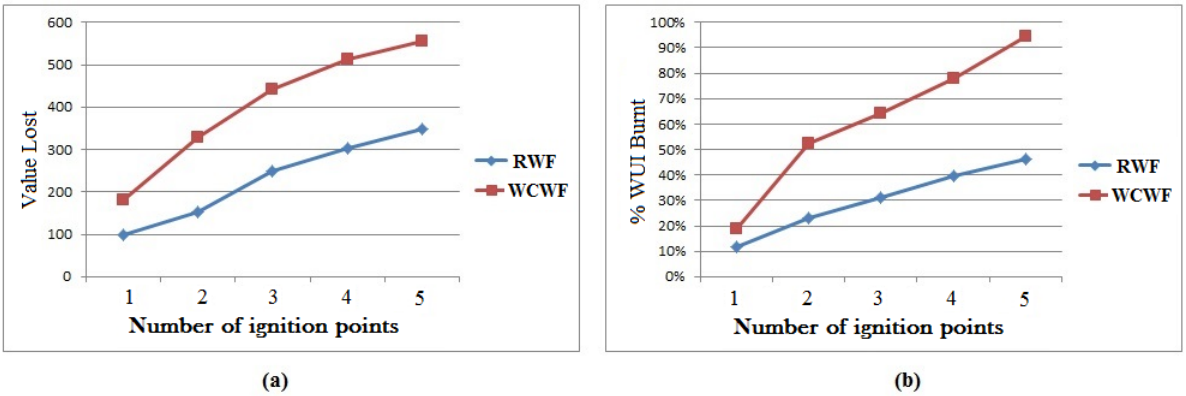

Fig. 7: (a) Value lost (unit-less) for the San Bernardino case with random wildfires (represented by RWF) and the worst-case wildfires (represented by WCWF) for different ignition point scenarios, (b) The percentage of WUI burnt in the San Bernardino case with random wildfires and the worst-case wildfires for different number of ignition point scenarios when fire last for 24 hours.

4. Discussion and conclusions

Wildfires can have serious and long-lasting impacts on ecological, social and economic systems [12]. It is necessary to identify and understand these impacts, and to develop cost effective mitigation strategies accordingly. In this paper, we studied the vulnerability of landscapes to wildfire threats considering the impact of fire ignition locations – the worst-case scenario. We compared the impacts of wildfires with optimally located ignition points (the worst-case wildfires) with the impacts of wildfires with randomly located ignition points (random wildfires). We used FlamMap to model fire behavior using landscape data, wind condition, and fuel moisture data, and developed an optimization model to find the maximum impact of wildfires and their optimal ignition locations. Three landscape test cases were used for experimentation and the impacts of various factors such as the number and location of ignition points, fire durations, and wind speeds were investigated. The proposed model is compact, and yet it can incorporate a variety of features such as the presence of fuel breaks and unburnable cells, and fire duration distribution.

The major contribution of this work is the development of a compact model for assessing the vulnerability of landscapes to wildfires regarding the location and number of ignition points – the worst-case scenario. The model is efficient and fast to solve. It takes less than a minute on a personal computer to solve the largest problem (San Bernardino landscape case) to optimality. The model can be used to assess the vulnerability of a landscape to wildfires under the worst-case consequence scenario. This assessment complements an assessment of the average-case consequence scenario. Thus far, researchers have focused on the average case and have not studied the worst case.

Our results suggest the worst-case wildfires cause more damage (more than two times on average) to the landscape test cases than random wildfires for both WUI and non-WUI landscapes. Although higher wind speed can exacerbate the impact of wildfires [53], our study shows that even under low wind speed conditions, the worst-case wildfires have higher impact on landscapes than random wildfires would have under high wind speed conditions. The worst-case wildfires spread faster and cause more damage in shorter period of time than random wildfires can cause in longer period of time. Within 12 hours, a worst-case wildfire with one ignition point can cause, on average, three times more damage to a landscape than a random wildfire with one ignition point.

For arson-induced wildfires, it is not only the location of ignition points that can be determined, but the number of ignition points is also part of the arsonists' decision process. Therefore, arson-induced wildfires can have more ignition points (multiple fires) than natural wildfires, which can make arson-induced wildfires more catastrophic and more difficult to suppress than natural wildfires. Our results indicate that the worst-case wildfires with five ignition points are 7 times more costly (in case of area burned) than random wildfires with one ignition point and 4 times more costly than random wildfires with two ignition points(Table 2). This difference can grow even larger if more ignition points are chosen in an arson-induced wildfire, which makes arson-induced wildfires even more catastrophic.

As illustrated in this research, the impact of worst-case wildfires can vary between different landscapes. This is likely due to differences in landscapes and vegetation characteristics that influence rate of spread, and major fire spread direction, both of which make a landscape more vulnerable to arson-induced wildfires. Our model can suggest high priority areas for wildfire risk mitigation planning, such as fuels treatment scheduling and fire suppression preparedness planning, to reduce the spread and intensity of potential worst-case wildfires and arson-induced wildfires. However, it should be noted that in reality, it is the land and fire managers who, based on their knowledge and expertise, make the ultimate decision.

There are several directions that future research extending this paper could take. First, one could extend this study and model the arson-induced wildfire problem as a Stackelberg game [54] model in which arsonists consider the possible mitigation response of fire managers and take the optimal action accordingly to minimize the mitigation effect. Another possible extension is to investigate the impact of arson-induced wildfires while also taking fire response into account, knowing how many resources and fire-response crews are available at various points in a landscape. This can be especially helpful in assessing the risk of arson-induced wildfires when adversaries are aware of fire response resources and their locations.

In this research we have developed a mathematical programming model to the combinatorially complex problem of landscape vulnerability assessments to arson-induced wildfires (worst-case wildfires). Our hope is that this study can begin to fill the gap in the literature, and assist landscape and wildfire managers in developing a fire management system resilient to potential arson-induced wildfire threats.

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Appendix

Table 6: The 95% confidence interval for percentages of landscapes burned by by random wildfires with X number of ignition points (represented by RWF(X)) for different number of ignition points, and fire duration scenarios

Fire	Landscape	RWF (1)		RWF (2)		RWF (3)		RWF (4)		RWF (5)	
Duration	Name	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
12 hours	Santa Fe	2.61	2.83	5.23	5.33	7.94	8.06	10.49	10.63	12.88	13.04
	Umpqua	2.26	2.54	4.73	4.87	7.12	7.28	9.35	9.63	11.58	11.78
	San Bernardino	4.42	4.66	9.23	9.38	13.54	13.73	17.53	17.74	21.35	21.59
18 hours	Santa Fe	4.90	5.34	11.26	11.46	16.36	16.60	21.15	21.41	25.63	25.89
	Umpqua	5.00	5.56	10.11	10.37	14.90	15.18	19.21	19.51	23.36	23.68
	San Bernardino	10.02	10.39	19.15	19.38	27.09	27.44	34.26	34.64	40.62	41.02
24 hours	Santa Fe	9.57	10.27	18.72	19.04	26.71	27.05	33.74	34.10	39.81	40.19
	Umpqua	8.84	9.72	17.25	17.63	24.60	25.00	31.15	31.57	36.60	37.00
	San Bernardino	16.81	17.31	30.79	31.09	41.87	42.38	51.24	51.78	58.75	59.29
	Average	7.16	7.63	14.05	14.28	20.01	20.30	25.35	25.66	30.06	30.39

Table 7: The 95% confidence interval for percentages of San Bernardino landscape burned by by random wildfires with X number of ignition points (represented by RWF(X)) for different number of ignition points, and under different fire duration and wind speed scenarios.

Fire	Wind	RWF (1)		RWF (2)		RWF (3)		RWF (4)		RWF (5)	
Duration	(MPH)	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
12 hours	8	3.77	4.06	7.58	7.76	11.08	11.28	14.50	14.72	17.76	17.99
	12	4.42	4.66	9.23	9.38	13.54	13.73	17.53	17.74	21.35	21.59
	16	5.75	6.00	11.43	11.59	16.54	16.77	21.34	21.60	25.91	26.18
18 hours	8	8.10	8.55	15.71	15.96	22.44	22.78	28.56	28.91	34.10	34.47
	12	10.02	10.39	19.15	19.38	27.09	27.44	34.26	34.64	40.62	41.02
	16	12.35	12.79	23.29	23.57	32.52	32.95	40.68	41.12	47.69	48.15
24 hours	8	13.69	14.23	25.49	25.77	35.19	35.66	43.36	43.83	50.45	50.94
	12	16.81	17.31	30.79	31.09	41.87	42.38	51.24	51.78	58.75	59.29
	16	20.48	21.15	36.90	36.90	49.09	49.68	58.97	59.56	66.57	67.15
Average		10.60	11.02	19.95	20.16	27.71	28.07	34.49	34.88	40.35	40.75

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