Optimal traffic calming: A mixed-integer bi-level programming model for locating sidewalks and crosswalks in a multimodal transportation network to maximize pedestrians' safety and network usability

Abstract

We study the effect that implementing traffic calming facilities has on the safety and usability of multimodal transportation networks. In particular, we examine the effect that installing sidewalks and crosswalks has on the total transportation cost and pedestrians' safety. A mathematical programming model is proposed for optimally locating sidewalks and crosswalks as traffic calming facilities in a transportation network with auto, public transit and walking as modes of transportation. The model's objective is to minimize the safety hazard for pedestrians and the total transportation cost of the network. We found that solving large instances of this problem using a commercial solver requires excessive computational resources. To improve the solution efficiency, we implemented a customized greedy heuristic and a simulated annealing algorithm. The computational results indicate that installing sidewalks and crosswalks at proper locations can reduce the overall transportation cost and improve pedestrians' safety. However, in low traffic conditions, this impact is smaller. The results also suggest that installing sidewalks and crosswalks can encourage walking and decrease use of cars, especially in high traffic congestion conditions. Surprisingly, the results do not suggest installing sidewalks and crosswalks at every possible location even when there is enough budget.

Keywords: Multimodal transportation network, mixed-integer nonlinear programming, network design, pedestrians' safety, traffic calming, user equilibrium.

1. Introduction

In many small communities in the United States, transportation is dominated by a single mode - the motor vehicle. Walking is usually not considered as a transportation mode; however, it is actually the most important mode of transportation in everyday life. It offers predictable travel time, continuous availability, while it is free, reliable, and non-polluting (Fruin, 1992). Yet, the field of transportation planning has little in the way of theory and methods for planning

walkable cities (Southworth, 2005). The lack of walkway infrastructures like sidewalks and crosswalks pose a safety hazard to pedestrians since they must walk along busy streets and highways to travel within the city. Since 1920s, there has been a growing concern about pedestrians' safety as pedestrians fatalities are a major part of all traffic fatalities (Campbell et al., 2004). The Federal Highway Administration (FHWA) estimated that 4,500 pedestrians are killed annually because of traffic accidents with motor vehicles, and as many as 88% of those accidents could have been avoided if walkways separate from travel lanes were available to pedestrians (FHWA, 2010). This implies the importance of pedestrians' safety in transportation network design for city planners and government. On the other hand, researchers have found that pedestrians are also concerned about safety and consider it as an important factor in transportation (Bahari et al., 2013; Seneviratne and Morrall, 1985; Weinstein Agrawal et al., 2008).

Several researches have shown the importance of traffic calming on safety (Bunn et al., 2003; Elvik, 2001; Huang and Cynecki, 2000; Lee et al., 2013; Retting et al., 2003). Traffic calming is the combination of mainly physical measures that reduces the negative effects of motor vehicle use, alters user behavior and improves conditions for non-motorized street users (FHWA and ITE, 1999). Traffic calming strategies can range from a few minor changes such as speed limits and speed humps applied to neighborhood streets to major rebuilding of street network (Drezner et al., 1999). In this study, we use traffic calming facilities such as sidewalks and crosswalks (we use S&C as the abbreviation for sidewalks and crosswalks in the rest of the paper) as pedestrians' infrastructures in designing a multimodal transportation network to enhance pedestrians' safety and increase network usability.

There are several aspects of this study that can contribute to the literature. Many transportation network design problems address only a single mode and "the literature of multimodal network design problem is very limited" (Farahani et al., 2013). Of the few existing studies in multimodal transportation problems, many studies assume no flow interaction between different transportation modes (Farahani et al., 2013). While in cases where public transit mode (e.g. trams, metros) uses exclusive lanes, the flow of the public transit mode has no effects on vehicle flows on roads, when the transportation modes share lanes, the flow of transit and auto modes do interact. Also, studies have ignored combined mode trips where travelers can use multiple modes along their trips as in park-and-ride, especially in the strategic level decisions

2

(Farahani et al., 2013). An important aspect of multimodal transportation systems with combined mode trips is to provide convenient mode transfer possibilities for travelers. Nowadays, with the advent of technologies like Uber, Lyft, SideCar and Curb, the combined mode trips seem more viable than ever before. The travelers now can switch between walking, transit and auto conveniently with no restriction as exists in park-and-ride. On pedestrian transportation literature, most studies are descriptive (Dill, 2004; Loeb and Clarke, 2009; Mitchell and MacGregor Smith, 2001; Randall and Baetz, 2001; Smith, 2001; Weinstein Agrawal et al., 2008), and to the best of our knowledge, no study has considered walking as a mode of transportation within a network design problem. Neither any study has included pedestrians' infrastructures in a transportation network design problem.

The objective in most studies in the transportation network design literature are primarily related to travel time (Fan and Machemehl, 2006a, 2006b; Lee and Vuchic, 2005; Mesbah et al., 2008; Resat and Turkay, 2015), or travel cost such as operator cost, user cost etc. (Cipriani et al., 2006; Fan and Machemehl, 2008; Gallo et al., 2011), and none of them addresses safety. In this study, we propose a network design framework for installation of S&C as traffic calming facilities to optimize the overall performance of a multimodal transportation network in terms of both mobility and safety. We develop a mathematical programming model for optimally locating these traffic calming facilities in a transportation network such that the overall transportation cost decreases and pedestrians' safety improves. We consider the flow interaction effects between the transportation modes: auto, transit and walking. We also allow travelers to switch their transportation modes along their trips (combined mode trips).

Assuming there is limited budget for city planners, and considering the large number of possible alternatives for locating and installing S&C, developing a multimodal transportation system regarding the optimal location of these city infrastructures is very challenging. Therefore, it is important to find the optimal locations for these infrastructures considering limited financial resources. In this study we model this problem as a bi-level network design problem. To reflect the importance of safety for both city planners and pedestrians, we include safety both in the design level (upper level) and in the user level (lower level) of the model. We found that solving large instances of this problem using a commercial solver requires excessive computational resources. Therefore, we apply two heuristic algorithms, a customized greedy heuristic and a simulated annealing algorithm. We test these algorithm on three problem instances: the

3

hypothetical Small network (a network with 4 nodes and 5 links), the Hearn network (Hearn and Ramana, 1998) and the Sioux Falls network (LeBlanc, 1975).

The goal of this study is to optimize the usability of the transportation system while ensuring safe travel for pedestrians by optimally locating S&C. Carefully installed S&C enhance walkability of transportation system which not only can improve pedestrians' safety, but can also encourage more people to walk (Gallimore et al., 2011; Pucher and Dijkstra, 2003; Staunton et al., 2003). In addition to health benefit of promoting walking, it can also help reduce vehicle miles traveled, alleviate traffic congestion, which in turn can help cut energy use and carbon emission and reduce noise and air pollution (Marshall and Garrick, 2010; Pucher and Dijkstra, 2003; Southworth, 2005). The major contribution of this research is to develop a quantitative network design model for locating traffic calming facilities in the transportation network to reduce the overall cost, improve pedestrians' safety and enhance walkability in a transportation network which can promote walking.

The remainder of the paper has the following outline. Section 2 describes the problem and presents the proposed mathematical model formulation for the problem. Section 3 discusses the solution methodologies for solving the problem. Section 4 conducts numerical experiments on three sample networks. Section 5 gives concluding remarks and discusses future research directions.

2. Optimal Traffic Calming Implementation in a Multimodal Transportation Problem (OTCIMTP)

The problem is, given a limited budget, where to install traffic calming facilities, S&C, in a transportation network, to minimize the total transportation cost, and improve pedestrians' safety. For any traffic calming installation layout, the problem also includes identifying a traffic assignment which is in accordance with the user equilibrium for a multimodal transportation problem. In this problem, travelers can use three modes of transportation: auto, transit and walking. Travelers are allowed to switch between these modes along their trips. We assume a traffic calming layout with the minimum transportation cost, and the minimum safety hazard for pedestrians, is the best traffic calming lay out.

We formulate OTCIMTP as a bi-level, leader-follower optimization model. In the upper level, the leader (city planners) decides where to locate the traffic calming facilities, and in the

4

lower level, the follower (travelers) decides on the travel route and mode of transportation (Farahani et al., 2013). The lower level problem is a traffic assignment problem under the user equilibrium.

We use a network reconstruction process to add S&C to an existing transportation network dataset with only one mode, the auto mode. Given the road transportation network dataset, this process adds walking and transit modes as a set of mode specific links and nodes to the transportation network. In this framework, pedestrians use walking links, auto passengers use auto links, and transit passengers use transit links. The reconstruction process is briefly discussed in the next section (for a more detailed discussion on the reconstruction process we refer the reader to (Parsafard et al., 2015)).

2.1. Network Representation

We propose a transportation network that consists of a network $\mathcal{G}(\mathcal{N}, \mathcal{L})$ made up of a set of links, $\mathcal{L}$, representing road segments, transit lines and sidewalks and crosswalks, and a set of nodes, $\mathcal{N}$, representing intersections between these links. Network $\mathcal{G}(\mathcal{N}, \mathcal{L})$ is reconstructed from a given auto-mode-only network $G(N, L)$. The original network $G(N, L)$ is made up of a set of links, L, representing only road segments, and a set of nodes, N, representing the intersections between the road segments. The original and the reconstructed networks for the Small network instance are illustrated in **Fig. 1**. The original network $G(L, N)$ in this example as illustrated in **Fig. 1(a)** consists of $N = \{1, 2, 3, 4\}$, representing nodes for the auto mode, and $L = \{(1,3), (1,4), (2,1), (2,3), (3,4)\}$, representing road segments (Parsafard et al., 2015).

The reconstruction mapping function $\mathfrak{R}$ is a process that takes the original network $G(N, L)$ and transforms it to a new network that in addition to road segments and their intersections also includes transit lines and S&C with separate links and nodes. For each link $l \in L$, representing a road segment in G, links l_{s1} and l_{s2} are added to the left and right hand sides of l in parallel, to respectively represent left- and right hand sidewalks for road segment l. Also, two crosswalk links $l_{c_{begin}}$ and $l_{c_{end}}$ are added one at the beginning and one at the end of each link l to the network. The crosswalk links cross the auto link l and connect the left- and right hand sidewalks. In addition to sidewalk and crosswalk nodes and links, transit nodes and links are also added to the network. To connect these walking (sidewalk and crosswalk) and transit nodes and links to

the original network, transfer links (representing the switch from one mode to another) are added to the network. For each origin and destination in the set of origin-destination pairs, dummy origin and destination nodes are added to the network. To ensure connectivity of the network, each dummy node is connected to its nearest sidewalk node using a connector link with zero travel time. This also ensures that travelers start and end their trips by walking (using sidewalk links). The reconstructed network for the Small network instance is illustrated in **Fig. 1 (b)**. In this figure, the dummy nodes are shown in green and are represented by 5, 6, 7 and 8. Walking nodes are represented with three digit numbers, e.g. 231. The transit nodes are represented with four digit numbers, e.g. 1001 (Parsafard et al., 2015).
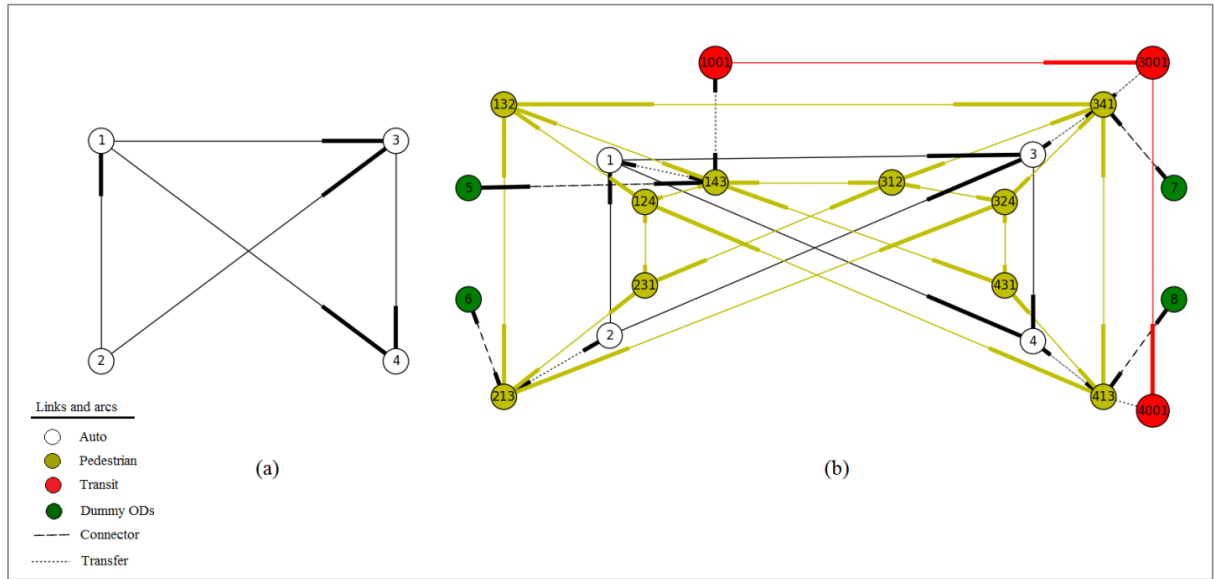

Fig. 1. Small network, (a) original network (b) reconstructed network.

2.2. Mixed integer bi-level programming formulation

In the proposed bi-level model, the upper level problem locates and installs S&C on the reconstructed network, and the lower level problem solves the corresponding traffic assignment problem and user equilibrium. The reconstructed network is represented with $\mathcal{G}(\mathcal{N}, \mathcal{L})$ in which $\mathcal{N}$ and $\mathcal{L}$ are the set of nodes and links. Links in $\mathcal{L}$ can be divided into six categories: A for auto links which is equal to set L in the original network $G(L, N)$, T for transit links, S for sidewalk links, C for crosswalk links, F for transfer links that connect walking links to auto and transit links, and R for connector links which connect the whole network together. The sets of incoming

and outgoing links to and from node $i \epsilon \mathcal{N}$ respectively are represented with $I(i)$ and $O(i)$. M is the set of transportation modes that contains indices 'a', 't' and 'w' representing auto, transit and walking modes, respectively. K represents the set of trips in the transportation network. For trip $k \in K$, d_k is the transportation demand, starting from the origin $\mathcal{O}_k$ to the destination $\mathcal{D}_k$. The set of traffic calming facilities (S&C) is represented with J. While we assume sidewalks ("s") installation include implementing both left- and right hand sidewalks, the begin- and end crosswalks ("c_{begin}" and "c_{end}") can be installed independently. If the set of traffic calming facilities available for link $l_a \in A$ (the subscript a indicates that link l is an auto link) is represented by J_{l_a}, then the decision for city planners to make is whether to install the traffic calming facility $j \in J_{l_a}$ that costs $c_{J_{l_a}}$ on the link l_a and imposes the transportation cost $\varphi_1(.)$. The criterion for this decision making is whether the overall transportation cost decreases while pedestrians' safety increases. This decision is limited to budget constraint on implementing traffic calming facilities (not exceeding b) on the transportation network. **Table 1** shows all the notations used in the proposed model.

As mentioned earlier, two traffic calming facilities are considered in this research, S&C. For crosswalks, we assume that there can be two crosswalk lines on each auto link, one at the beginning (represented by $l_{c_{begin}}$), and one at the end (represented by $l_{c_{end}}$). We assume that the begin- and end-crosswalk can exist and work independently. But for sidewalks, we assume that the left- and the right-hand sidewalks are dependent and cannot exist independently, meaning that if we build a sidewalk for a road, we build it for both sides of the road.

The other main assumptions of the model are the followings:

a) As walking is considered a mode of transportation, the travel demand is based on the number of travelers, not the number of autos or transits. There are three different traveler types: auto travelers, transit travelers and pedestrians. We also assume that each transit carries 20 travelers which is equivalent to 4 autos (Aashtiani, 1979).

b) The transit schedule is considered as a transfer cost for pedestrians who take transit and change their mode from walking to transit. The same is true for auto (Uber waiting time).

Table 1. Notations

Sets and Indices

N:	Set of nodes indexed by $i = 1, \dots, N$.
L:	Set of links, indexed by $l = 1, \dots, L$ or (i, i').
$\mathcal{N}$	Set of nodes in the reconstructed graph indexed by $i = 1, \dots, \mathcal{N}$.
$\mathcal{L}$	Set of links in the reconstructed network, indexed by $l = 1, \dots, \mathcal{L}$ or (i, i').
$I(i)$	Set of incoming links to node i.
$O(i)$	Set of outgoing links from node i.
M:	Set of transportation modes, denoted by $m = a, t, w$ where "a", "t", and "w" represent auto, transit, and walking modes respectively.
J:	Set of traffic calming facilities indexed by $j = c_{begin}, c_{end}, s$, where "$c_{begin}$", "$c_{end}$", and "$s$" are begin-crosswalk, end-crosswalk and sidewalk respectively
J_{la}:	Set of traffic calming facilities available on auto link la.
K:	Set of trips indexed by k.

Parameters and Functions

b:	The budget (\$)
ϑ:	Value of time (\$/h)
ω:	transit passenger equivalent factor
θ:	auto passenger equivalent factor
τ_{l_f}:	Transfer cost for switching between walking mode to auto or transit modes (the transfer cost is different for auto and transit).
σ	Average cost of a pedestrian crash in dollars (\$)
t_{l_m}:	Free-flow travel time for link l_m
$d_k, \mathcal{O}_k, \mathcal{D}_k$	Demand, origin and destination of trip k
γ_{l_m}:	Capacity of link l_m
δ:	Safety weight factor that quantifies the travelers' preference between time delay and safety.
$\varphi_l(.)$:	Travel cost function for link l in the upper level problem
$\phi_l(.)$:	Travel cost function for link l in the lower level problem
$\psi(X_{l_s})$	The probability of a pedestrian getting into a crash when walking along the auto link adjacent to the sidewalk link l_s
$P_{la}(X_{l_s})$	Pedestrians crash probability function on auto link la
$\alpha_1, \alpha_2, \beta_1, \dots, \beta_6$	Multipliers and powers used in the objective function formulation

Variables

$\pi_{i,k}$:	Auxiliary variable (the dual variables of the corresponding shortest path problem)
$X_{k,l}$:	Number of trip k flows on link l
X_l:	$(X_{k,l})$, the vector of flow variables for all trips
X:	The vector of all flow variables on all links for all trips
$Y_{l,j}$:	1 if traffic calming j is implemented on auto link l, 0 otherwise
Y	$(Y_{l,j})$, the vector of traffic calming variables

c) The problem is deterministic, so the transportation demands and supplies are known and there is no randomness in road capacity either.

d) The demands for modes are not fixed and not known in advance, and the travelers decide their mode of transportation based on their traffic disutility function.

We formulate OTCIMTP as a link-based model. Each link in the transportation network has a transportation cost that includes the travel time converted to a dollar value.

2.2.1 Travel cost function

For auto links, $l_a \in A$, the travel cost function is as follows:

$$\varphi_{l_a}(X,Y) = \left(t_{l_a}\left(1 + \alpha_1 \left(\frac{\sum_{k \in K} X_{k,l_a} + \sum_{k \in K} \omega X_{k,l_t}}{\gamma_{l_a}} \right)^{\beta_1} \right) \right. \tag{i}$$

$$+ \left(\left(\frac{\sum_{k \in K} X_{k,l_{s1}}}{\gamma_{l_{s1}}} \right)^{\beta_2} + \left(\frac{\sum_{k \in K} X_{k,l_{s2}}}{\gamma_{l_{s2}}} \right)^{\beta_2} \right) \times \left(1 - y_{l_a,s1} \right) \tag{ii}$$

$$\left. + \left(\frac{\sum_{k \in K} X_{k,l_{c_{begin}}}}{\gamma_{l_{c_{begin}}}} \right)^{\beta_3} \times \left(y_{l_a,c_{begin}} \right) + \left(\frac{\sum_{k \in K} X_{k,l_{c_{end}}}}{\gamma_{l_{c_{end}}}} \right)^{\beta_3} \times \left(y_{l_a,c_{end}} \right) \right) \times \vartheta + \mu_{l_a} \quad (iii) \tag{1}$$

The travel cost of auto on link $l_a \in A$ is affected by (a) the amount of flow on auto link l_a, (b) the amount of flow on the other links associated with link l_a (transit and walking modes), and (c) whether traffic calming facilities, S&C, are installed on link l_a. To incorporate these three factors into the travel cost of auto links, we divide the auto travel cost into three parts. We use the coefficient ϑ (the value of time) to convert all these travel costs to a dollar value.

Part (i) reflects the effect of traffic flows of auto in conjunction with the flows of transit. It is a BPR[1]-like function that includes the flow of auto as well as the flow of transit on auto link l_a. Note that transit and auto share the same lane. γ_{l_a} is the capacity of link l_a. The effect of transit flow on the auto travel cost function on link l_a includes a transit passenger equivalent factor ω. This factor is to compromise the difference between the capacity for passengers in a transit

---

[1] U.S. Bureau of Public Roads

versus an auto. t_{l_a} is the free flow travel time on link l_a. The quantities α_1 and β_1 are model parameters. μ_{la} is the out-of-pocket cost (the indirect costs) that auto drivers have to pay (such as gas, insurance etc.) for using the auto link l_a.

Part (ii) reflects the effect of traffic flows on the left- and right-hand sidewalks on auto link l_a. It is a function of flows in the sidewalks over their capacities raise to power β_1. $X_{k,l_{s1}}$ and $X_{k,l_{s2}}$ are the amount of flows in the left- and right-hand sidewalks respectively, and $\gamma_{l_{s1}}$ and $\gamma_{l_{s2}}$ are the capacity of the left- and right hand sidewalks respectively. To include the effect of installing sidewalks on the travel cost of auto link l_a, the associated decision variable $y_{l_{a,s1}}$ is included. When a sidewalk is installed, we assume that both the left- and the right-hand sidewalks are built on the sides of a street. So when $y_{l_{a,s1}} = 1$, two sidewalks are installed on both sides of the street represented by the auto link l_a. We assume that the flow of pedestrians on installed sidewalks do not influence the flow of the adjacent auto links. Otherwise, in the absence of installed sidewalks, we assume that pedestrians have to walk on the sides of streets and share the road with autos. This affects the travel cost of autos in return. Thus, installing sidewalks on auto links create separate walkways for pedestrians. This can decrease the travel cost of auto, as the flow of pedestrians no longer influences the travel cost of autos. This is reflected in the auto travel cost function shown in equation (1) by the term $(1 - y_{l_a,s1})$. If sidewalks are installed on auto link l_a, the corresponding binary variable $y_{l_a,s}$ equals 1 which in turn causes the travel cost function in part (i) in equation (1) equals to zero. The quantity β_2 is a model parameter.

Part (iii) reflects the effect of traffic flows on crosswalks, both the begin- and end-crosswalks on auto link l_a. It is a function of flows in the crosswalks over their capacities raise to power β_2. $X_{k,l_{c_{begin}}}$ and $X_{k,l_{c_{end}}}$ are the amount of flows in the begin- and end crosswalks respectively, and $\gamma_{l_{c_{begin}}}$ and $\gamma_{l_{c_{end}}}$ are the capacity of these sidewalks. To include the effect of installing crosswalks on the travel cost of auto link l_a, the associated decision variables $y_{l_{a,c_{begin}}}$ and $y_{l_{a,c_{end}}}$ are included. We assume the begin- and end-crosswalks can be implemented separately and independently (as opposed to sidewalks). Installing begin- or end-crosswalks can encourage more pedestrians to cross the auto link l_a and increases its travel cost. This is reflected in part (ii) of equation (1). When a crosswalk is installed ($y_{l_{a,c_{begin}}}$ or $y_{l_{a,c_{end}}} = 1$), the flow of pedestrians affect the travel

cost of auto on link l. Otherwise, if no crosswalk is installed, then $y_{l_{a,c_{begin}}} = y_{l_{a,c_{end}}} = 0$, and part (ii) in equation (1) equals to 0. The quantity β_3 is a model parameter.

The travel cost function for transit links $\varphi_{l_t}(.)$ is as follows:

$$\varphi_{l_t}(X,Y) = \vartheta \times t_{l_t}\left(1 + \alpha_2\left(\frac{\sum_{k \in K}(X_{k,l_t} + \theta X_{kl_a})}{\gamma_{l_t}}\right)^{\beta_4}\right) \tag{2}$$

This is a BPR-like function that includes the flow of transit as well as the flow of auto on link l_t. The effect of pedestrian flows on S&C are included in the auto travel cost (equation (1)), which in turn affects the flow of autos, and since the effect of auto flows is reflected in $\varphi_{l_t}(X,Y)$, therefore, equation (2) also reflects the indirect effect of pedestrian flows. The effect of auto flow on the transit travel cost function on link l_t includes an auto passenger equivalent factor θ. This factor is used to compromise the difference between the capacity for passengers in a transit versus an auto. t_{l_t} is the free flow travel time on link l_t. The quantities α_2 and β_4 are model parameters. We use the coefficient ϑ (the value of time) to convert the travel cost to a dollar value.

The travel cost function for crosswalk links $\varphi_{l_c}(.)$ is also a BPR-like travel cost function as follows:

$$\varphi_{l_c}(X,Y) = \vartheta \times t_{l_c}\left(1 + \alpha_3\left(\frac{X_{l_c}}{\gamma_{l_c}}\right)^{\beta_5}\right) \tag{3}$$

In this formula, t_{l_c} is the free flow travel time for crossing an street on link l_c. The free flow travel time can be found based on the length of l_c and walking speed. X_{l_c} is the flow of pedestrians crossing an street using the crossover link l_c, which has the capacity of γ_{l_c}. The quantities α_3 and β_5 are model parameters. We use the coefficient ϑ (the value of time) to convert the travel cost to a dollar value.

The travel cost function for sidewalk links $\varphi_{l_s}(.)$ is as follows:

$$\varphi_{l_s}(X,Y) = \vartheta \times t_{l_s}\left(1 + \alpha_4\left(\frac{X_{l_s}}{\gamma_{l_s}}\right)^{\beta_6}\right) \tag{i}$$

$$+ \left(1 - y_{l_s}\right) \times \sigma P_{l_a}\left(X_{l_a}\right) X_{l_s} \qquad\qquad (ii) \qquad (4)$$

This travel cost function consists of two parts: Part (i) reflects sidewalks' travel time, and part (ii) reflects pedestrians' safety on sidewalks. As we mentioned earlier in the introduction, safety is an important factor for pedestrians. According to a pedestrian survey in 2006 (Weinstein Agrawal et al., 2008), 99% of respondents rated "choosing the shortest route" the most important factor in choosing their travel route, while 87% rated "having sidewalks in good condition" as their most important factor. Therefore, we include both travel time and safety into the travel cost function for pedestrians on sidewalks. Since travel time is in time unit and safety is in dollar unit, we convert the travel time part to a dollar value using the multiplier ϑ.

Part (i) is a BPR like function that consists of the flow of pedestrians (X_{l_s}) and the capacity (γ_{l_s}) of link l_s. In this formula, t_{l_s} is the free-flow travel time of link l_s. The quantities α_4 and β_6 are model parameters.

Part (ii) is the safety part and computes the expected cost of pedestrians' crashes (as a penalty cost in absence of installed sidewalks) on the auto link l_a adjacent to sidewalk l_s. When no sidewalk is installed, pedestrians have to walk along the streets which is unsafe. To compute the expected cost of pedestrians' crashes, we multiply the probability that a given pedestrian will get in a crash ($P_{l_a}\left(X_{l_a}\right)X_{l_s}$) by the average cost of a pedestrian crash (σ) (Gårder, 2004). The term $P_{l_a}\left(X_{l_a}\right)$ is the pedestrian crash probability function when pedestrians walk along the auto link l_a. This function is obtained by dividing the total number of crashes on a given road segment by the total traffic flow on that segment. Then by using a simple linear regression among all the streets, we calculate the crash probability function. For this regression we used the historical crash data from Starkville, Mississippi, U.S.A. (see [33] for more details).

In equation (4), to incorporate the effect that installing sidewalks has on the pedestrians' safety, the associated decision variable y_{l_s} is used. In the case of an installed sidewalk ($y_{l_s} = 1$), the cost associated with pedestrians' crashes in the safety part (part (ii)) becomes 0. However, when no sidewalk is installed ($y_{l_s} = 0$), part(ii) (which is not equal to 0) enforces the cost of pedestrians' crashes due to absence of sidewalks.

For sidewalk links, the travel cost function for the upper level problem (equation (4)) is slightly different from the travel cost function in the lower level. The travel cost function for the lower level is

$$\acute{\varphi}_{l_s}(X,Y) = \quad \vartheta(1-\delta) \times t_{l_s}\left(1 + \propto_4 \left(\frac{X_{l_s}}{\gamma_{l_s}}\right)^{\beta_6}\right) \qquad\qquad (i)$$

$$+ \delta\left(1 - y_{l_s}\right) \times \sigma\psi\left(X_{l_s}\right) \qquad\qquad (ii) \qquad (5)$$

The travel cost function for the lower level also consists of two parts: Part (i) reflects sidewalks' travel time, and Part (ii) reflects pedestrians' safety on sidewalks. Part (i) is the same BPR function used in equation (4). However, we use an adjustment safety weight factor, δ, to combine the travel time part and the safety part for the travel cost in the lower level. The adjustment weight factor can be used by city planners for finding a balance between travel safety and travel time when designing a transportation network.

In the travel cost function for the lower level, part (ii) computes the expected cost for an individual when walking along the auto link l_a (adjacent to the sidewalk l_s) for which the sidewalk has not been built. In this formula $\psi\left(X_{l_s}\right)$ is the probability that a pedestrian gets into a crash when walking along the auto link l_a (due to lack of an installed sidewalk) and is computed as follows:

$$\psi\left(X_{l_s}\right) = \left(\frac{P_{l_a}\left(X_{l_a}\right)X_{l_s}}{0.01 \times \gamma_{l_s}}\right) \qquad\qquad (5a)$$

The numerator in (5a) computes the expected number of pedestrians crashes on auto link l_a when pedestrians walk along the street. The denominator in (5a) is an estimate of the average number of pedestrians walking along auto link l_a. We assume the average number of pedestrians walking along a street is 1% of the capacity of the adjacent sidewalk if it was built.

Similar to equation (4), to incorporate the effect that installing sidewalks has on the pedestrians' safety, the associated decision variable y_{l_s} is used in equation (5). In case of an installed sidewalk (when $y_{l_s} = 1$), the cost associated with pedestrians crashes in the safety part (part (ii)) becomes 0. However, when no sidewalk is installed (when $y_{l_s} = 0$), part(ii) (which is not equal to 0) enforces the cost of pedestrians' crashes due to absence of sidewalks.

The links that connect the pedestrian links to auto and transit links are called "transfer links". The travel cost function for transfer links $\varphi_{l_f}(.)$ is as follows:

$$\varphi_{l_f}(X,Y) = \vartheta \times \tau_{l_f} \qquad (6)$$

We assume that the transfer cost on each transfer link is a constant value which indicates the total walking time to reach the transfer station and the total waiting time in the transfer station. There are two types of transfer links: auto-walking and transit-walking transfer links. In order to transfer from transit to auto (or auto to transit), both of these transfer links must be used. This total time is converted to a dollar value using the value of time factor ϑ.

And finally, the travel cost function for connector links $\varphi_{l_r}(.)$ is as follows:

$$\varphi_{l_r}(X,Y) = 0 \qquad (7)$$

Since the connector links are hypothetical, their travel cost is considered 0 (as we assumed).

The objective function in the upper level and the lower level are different. This is due to incorporating pedestrians' safety on sidewalks into the objective of the transportation problem in the upper level (design level) problem. However, pedestrians' safety is also incorporated into the objective function of the lower level (user level) problem. We assume that the travel cost for all links, except sidewalk links, is the same for the upper level problem and the lower level problem:
$$\acute{\varphi}_l(X,Y) = \varphi_l(X,Y) \quad \forall\ l\epsilon\mathcal{L}\backslash\{l_s|s \in\ S\}.$$

2.2.2 Mathematical formulation for OTCIMPT:

The proposed mathematical model is formulated as follows:

$$Min \sum_{k\in K} \sum_{l\in L} \varphi_l(X,Y)X_{k,l} \qquad (8)$$

$$s.t.$$

$$X_{k,l}\left(\acute{\varphi}_l(X,Y) - \left(\pi_{i',k} - \pi_{i,k}\right)\right) = 0 \qquad \forall\ l = (i,i')\epsilon\mathcal{L}, k\epsilon K \qquad (9)$$

$$\sum_{l \in O(\mathcal{O}_k)} X_{k,l} = d_k \qquad\qquad \forall\, k \in K \qquad\qquad (10)$$

$$\sum_{l \in I(i)} X_{k,l} \;-\; \sum_{l' \in O(i)} X_{k,l'} = 0 \qquad\qquad \forall\, i \epsilon \mathcal{N} \setminus \{\mathcal{O}_k, \mathcal{D}_k\}, k \epsilon K \qquad\qquad (11)$$

$$\sum_{j \in J} \sum_{l \in L} c_{j,l} y_{l,j} \leq b \qquad\qquad\qquad\qquad (12)$$

$$X_{k,l} \geq 0 \qquad\qquad \forall\, k \epsilon K, l \epsilon \mathcal{L} \qquad\qquad (13)$$

$$\pi_{ik} \geq 0 \qquad\qquad \forall\, k \epsilon K, i \epsilon I \qquad\qquad (14)$$

$$y_{l,j} \epsilon \{0,1\} \qquad\qquad \forall\, j \epsilon J, l \epsilon L \qquad\qquad (15)$$

The objective function (8) is to minimize the total transportation cost, including travel time and cost due to lack of pedestrians safety, in the network (in the objective function, lack of safety is penalized equivalent to a dollar value, therefore, by minimizing total cost we are also minimizing lack of safety). Constraints (9) enforce the optimal flow solution to be at travel cost equilibrium. Constraints (10) require that all of the demand flows through the network for every trip. Constraints (11) enforce conservation of flow for all nodes in the network. Constraint (12) is the budget constraint, and constraints (13) to (15) serve to restrict the range of variables.

Though the problem as a whole is a (non-convex) mixed-integer nonlinear programming problem, the following proposition shows that the lower level problem is convex.

Proposition 1: The objective function of the lower level problem, $\varphi_l(.)$, is convex.

Proof: See Appendix. □

For any solution to the upper level problem which locates and installs S&C, we use the nonlinear complementary algorithm (Aashtiani, 1979) to solve the traffic assignment and user equilibrium in the lower level. The nonlinear complementary algorithm does not require the lower level problem to be convex. However, since the problem is convex, any other algorithm that requires convexity can also be used.

3. Methodology

Solving a bi-level network design problem is difficult as the problem is NP-hard. In fact, as Ben-Ayed and Blair (Ben-Ayed and Blair, 1990) showed, even a linear bi-level problem (or bi-level linear problem; BLP) is NP-hard. Therefore, solving the problem for large scale instances using exact solution methods use extensive computational resources. Studies on multimodal network design problems are very few and the solution techniques used in these studies are mostly approximate methods (Farahani et al., 2013). A single level formulation for the problem is more tractable (Farvaresh and Sepehri, 2011), however, not always possible. Thus, heuristic algorithms are often developed for transportation network design problems (Farahani et al., 2013). Meta-heuristic algorithms are also very common in transportation network design problems e.g. (Drezner and Salhi, 2002; Miandoabchi et al., 2011a, 2011b; Yamada and Febri, 2015; Yang et al., 2007), especially for large scale problems. A major benefit of these methods is that they run much faster than exact methods. Some common methods of this kind include Genetic Algorithm, Simulated Annealing, Tabu Search and Ant Colony Optimization (Farahani et al., 2013) and matheuristic, a combination of mathematical programming and metaheuristics (Brouer et al., 2014).

To solve the proposed bi-level transportation network design problem in this paper, we use an exact approach and two heuristic ones. For the exact approach we implement the model in YALMIP (version 20141030) (Lofberg, 2004) and solve it using the BARON solver (version v1.69) (Sahinidis, 2014), a computational system that can solve mixed integer nonlinear programming problems. We also develop a customized greedy heuristic (GH) and a simulated annealing (SA) algorithm. These algorithms are used to first solve the upper level problem, where to install S&C in the network considering the limited budget. Then a nonlinear complementary algorithm (Aashtiani, 1979) is used to solve the lower level problem (the user equilibrium traffic assignment problem on the reconstructed network). We use a link-list dynamic data structure proposed by (Toobaie et al., 2010), which was reported to outperform the Frank-Wolfe algorithm (Frank and Wolfe, 1956). The advantages of the nonlinear complementary algorithm are its speed and the fact that it allows for a general cost function (i.e. the travel cost is a function of all the flows in the network). In the following we describe in more details the customized greedy heuristic and the simulated annealing algorithm proposed for this problem.

3.1. Greedy Heuristic

The greedy heuristic (GH) is a simple heuristic algorithm that makes the locally optimal choice at each stage with the hope of finding a global optimum. The GH in this study starts with a null solution (no sidewalks or crosswalks installed) and iteratively finds a new solution suggesting where to install a new sidewalk or crosswalk until the budget is exhausted. For a given solution x, the nonlinear complementary algorithm is used to compute its corresponding objective value, $f(x)$. The benefit-cost ratio $\frac{f(x)}{Cost(x)}$ is computed to evaluate the merit of that solution. The solution that has the largest benefit-cost ratio in that iteration is accepted (i.e. the corresponding sidewalk or crosswalk is installed). Additional S&C are installed at successive iterations in a similar fashion until the budget is exhausted.

In this study, we assume that the cost of installing S&C at any location is the same. Therefore, instead of $\frac{f(x)}{Cost(x)}$ we can use $f(x)$ in evaluating solution x.

3.2. Simulated Annealing

Simulated annealing (SA) is a probabilistic metaheuristic that emulates the physical gradual cooling process that produces high quality crystals. The method was proposed by Kirkpatick and Vecchi (Kirkpatrick and Vecchi, 1983). Variant of SA algorithms have been successfully applied to different optimization problems (Aliakbarian et al., 2015; Kia et al., 2012; Madadi et al., 2014; Miandoabchi et al., 2013).

An SA algorithm repeats an iterative neighbor generation procedure and follows search directions that improve objective function value. To escape from local optima, the SA algorithm offers the possibility to accept worse solutions with a probability that decreases as the algorithm moves toward completion. In each iteration, the difference between the objective value of the current solution $f(x)$ and the new solution $f(\acute{x})$ is evaluated as $\Delta = f(x) - f(\acute{x})$. If $\Delta \geq 0$ (for a minimization problem) the new solution $\acute{x}$ is accepted, otherwise it will be accepted with a probability of $p = \exp\left(\frac{\Delta}{T}\right)$, in which T is a parameter called the temperature of the current state. The factors that influence acceptance probability are the degree of objective function value degradation Δ, as well as the temperature T. Smaller degradation and higher temperature induce higher acceptance probability. The temperature can be controlled by a process called the cooling

schedule, which specifies how it should be progressively reduced to make the procedure more selective as the search progresses to neighborhoods of good solutions (Bouleimen and Lecocq, 2003).

The cooling schedule starts with a high temperature T_{max} so that it allows acceptance of new neighbor solutions with higher probability. An attenuation factor $\propto (0 < \propto < 1)$ is used to decrease the temperature in each iteration, so the acceptance probability decreases. The algorithm is terminated when the current temperature reaches the minimum temperature (T_{min}).

Simulated Annealing

1   Initialization: generate a random solution
     $x \leftarrow RandomSolution()$
2   Solve the traffic assignment problem at equilibrium for each link:
     $(flow, cost) \leftarrow NonLinearComplementary(x)$
3   Evaluate the objective value for the current solution:
     $f(x) \leftarrow EvaluateObjVal(flow, cost)$
4   $T \leftarrow T_{max}$
5   **while** $(T > T_{min})$ **do**
6      $i \leftarrow 1$
7   **while** $(i < iter_{max})$ **do**
8         $x' \leftarrow FindNeighborSolution(x)$
9         $(flow, cost) \leftarrow NonLinearComplementary(x')$
10        $f(x') \leftarrow EvaluateObjVal(flow, cost)$
11        $\Delta \leftarrow (f(x) - f(x'))$
12   **if** $(\Delta < 0)$ **then**
13            $BoltzmannValue \leftarrow \exp\left(\frac{-\Delta}{T}\right)$
14   **if** $(BoltzmannValue \leq Random(0,1))$ **then**
15   Accept the new solution: $x \leftarrow x'$
16   **end if**
17   **else**
18   Accept the new solution: $x \leftarrow x'$
19   **end if**
20        $i \leftarrow i + 1$
21   **end while**
22      $T \leftarrow T * \propto$
23   **end while**

Fig. 2. The pseudo code for the proposed SA

Table 2. SA parameters and their values

Parameter	Value
T_{max}	42000
T_{min}	0.00001
$Iter_{max}$	20
$\propto$	0.80

The neighborhood search used in in this paper comprises of two steps: (1) removing an installed traffic calming and (2) installing a traffic calming. For a given solution, we first

select an auto link arbitrarily from the pool of auto links with installed traffic calmings, and uninstall its traffic calming (if more than one traffic calming is installed on that auto link, one is arbitrarily selected and uninstalled). Then, we update the budget (the cost of the traffic calming that was uninstalled is reimbursed). Then we arbitrarily select another auto link (from the pool of all auto links) and install a traffic calming (either sidewalk or crosswalk, arbitrarily chosen) on that auto link. If the selected auto link already has that traffic calming installed, we discard that link and select another one. **Fig. 2** Shows the pseudo code for the proposed SA, and **Table 2** shows the tuned parameters used in the cooling schedule.

4. Numerical Experiments

Three sample networks are used for experimentation, a small hypothetical network (called "Small network"), the Hearn network (Hearn and Ramana, 1998) and the Sioux Falls network. The characteristics of these networks are given in **Table 3**. For the Sioux Falls network, the demand for each origin-destination pair is available. However, for the Small and Hearn networks, which are hypothetical transportation network instances, no such data are available. Therefore, we generate these data arbitrarily for experimentation. **Table 4** shows the arbitrarily-generated demand data that we use in this paper. **Table 5** shows the parameters we use in the travel cost functions that was presented in section 2.

Table 3. Sample Transportation Networks

Network	Num. of OD pairs	Original network		Reconstructed Network	
		Num. of nodes	Num. of links	Num. of nodes	Num. of links
Small	4	4	5	21	65
Hearn	4	9	18	55	192
Sioux Falls	552	24	76	143	523

Table 4. Demand for different OD pairs for the Small and Hearn Networks

Small Network			Hearn Network		
OD Pairs		Demand	OD Pairs		Demand
from	to		from	to	
5	7	10	10	12	20
5	8	40	10	13	40
6	7	20	11	12	60
6	8	60	11	13	80

Table 5. Parameters' values in travel cost functions used in this study

Parameter	Value	Description
α_1	0.15	Coefficient of the BPR like function in (1)
α_2	0.15	Coefficient of the BPR like function in (2)
α_3	2	Coefficient of the BPR like function in (3)
α_4	2	Coefficient of the BPR like function in (4) and (5)
β_1	4	Power in equation (1)
β_2, β_3	2	Power in equation (1)
β_4	4	Power in equation (2)
β_5	2	in equation (3)
β_6	2	in equation (4) and (5)
ω	0.2	Transit to auto passenger equivalent factor in equation (1)
θ	5	Auto to transit passenger equivalent factor in equation (2)
τ	3, 5	Transfer cost for walking mode to auto and transit modes respectively and vice versa (6)
σ	78300	Average cost of a pedestrian crash in dollar (4), (5)
ϑ	0.33	Value of time in (1)-(6)

The parameters used as coefficients in the BPR functions in **Table 5** are commonly used values. It is assumed that each transit carries 20 passengers, 4 times more than what a car is assumed to carry (Aashtiani, 1979). This makes $\omega = \frac{4}{20} = 0.2$, and $\theta = \frac{20}{4} = 5$ as shown in **Table 5**. The average cost of pedestrian crash ($\sigma = \$78300$) is computed using data from (Gårder, 2004). Highway Economic Recruitments System (HERS) (Vandervalk et al., 2014) considers $19.86 as the value of time (per hour), which is equal to $ 0.33 per minute. Therefore we assume that $\vartheta = 0.33$ (the value of time). The coefficients of the linear regression model used in the pedestrian crash probability function, $P_{la}(X_{ls})$, in travel cost function presented in equations (11) and (12) are 1.7^{-7} and 3.59^{-10} for the interception and slope respectively. Therefore $P_{la}(X_{ls}) = 1.7^{-7} X_{ls} + 3.59^{-10}$ (for more details we refer the reader to (Parsafard et al., 2015)).

4.1. Computational comparison

We used the BARON solver to solve the mathematical model. After running for 24 hours, BARON did not provide any feasible solution, even for the Small network. On the other hand, the GH and the SA, produced competitive solutions in a reasonable time (as shown in **Table 5**).

Table 5 shows the computational results, and the solutions produced by SA and GH algorithms after installing S&C in the transportation networks. As it can be seen in the table, the two algorithms produced solutions of similar quality. However, SA is slightly better for the larger Sioux Falls network. SA is also much faster for the Sioux Falls network; however, it is slower for the Small and Hearn networks. As shown in **Table 5**, the results for all three networks suggest that carefully installing S&C reduces the overall transportation cost, however, the cost reduction is more significant in the Sioux Falls network, with 59 % decrease in the overall cost (solution produced by the SA).

Table 5. A comparison of the computational performance of SA and GH

(% Reduction in cost $= \frac{f(x^0)-f(x^*)}{f(x^0)}$, x^0: The null solution i.e. when no S&C are installed,

x^*: Solution found by our algorithms).

Networks	% Reduction in cost		Time (sec)	
	SA	GH	SA	GH
Small	% 12	% 12	20.7	0.92
Hearn	% 5	% 5	31	2.6
Sioux Falls	% 59	% 58	199	504

The average demand per OD pair for the Small, Hearn and the Sioux Falls networks are 33, 50, and 653 respectively and the average demand per link are 2, 1 and 689 respectively.

We speculate that the higher demand per link and per OD pair in the Sioux Falls network might be the reason for the higher reduction in cost for this network. We will investigate the impact of demand for these networks later in section 3.4. Regarding the computation time of the two algorithms, as shown in **Table 5**, for the hypothetical Small and Hearn networks, GH is faster than SA; however for the Sioux Falls network SA is faster. To investigate the cause of these differences, we counted the number of times that the nonlinear complementary algorithm is called by these two algorithms (for solving the user equilibrium in the lower level problem). We learnt that for the Small and Hearn networks, the nonlinear complementary algorithm is called more often in the SA than the GH. For the Sioux Falls network, however, as the budget increases, the nonlinear complementary algorithm is called more often in the GH than the SA. Therefore, the GH becomes computationally more expensive and less efficient than SA for the larger Sioux Falls network (**Fig. 3**). In summary, for the larger transportation network (the Sioux Falls), the SA outperforms GH both in solution quality and computation time.

As shown in **Table 5**, installing S&C reduces the total transportation cost. To test the robustness of these results, we do a sensitivity analysis. We study the impact of S&C on the total transportation cost under different conditions such as different budget for installing S&C, different demand in the transportation networks, and different values for safety weight factor.
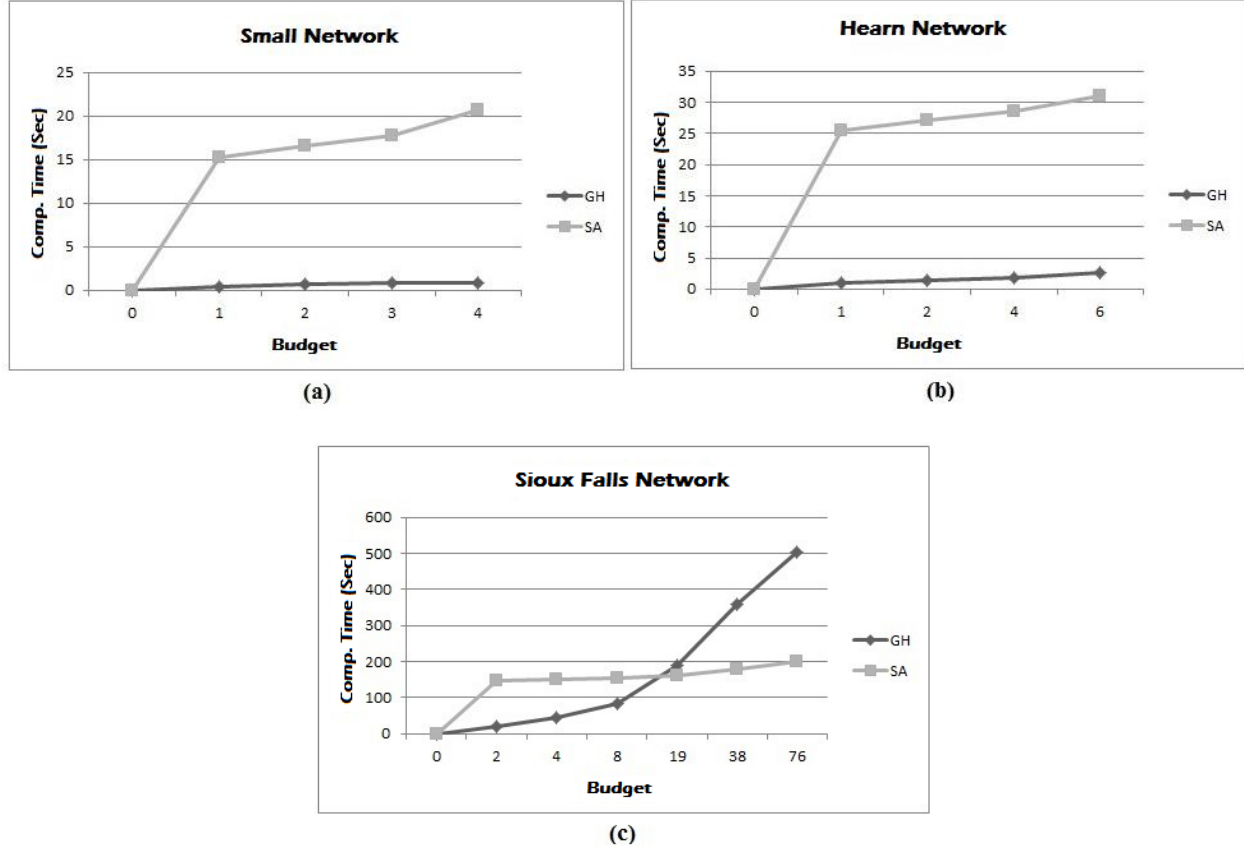

Fig.3. Computation time, GH vs. SA for (a) Small network, (b) Hearn network and (c) Sioux Falls network

4.2. Sensitivity analysis: Budget

In the previous experiment (**Table 5**), we assumed that the budget for installing S&C was unlimited. To test the performance of GH and SA with limited budget, we ran another set of experiments imposing a limit on the budget for installing S&C. Our results show that as the budget increases and more S&C are installed, the overall transportation cost decreases in all three networks; however, for the Sioux Falls network this change is more significant. For all three networks, as budget increases the degree by which the overall cost decreases diminishes and there exists a point where adding more S&C no longer affects the overall cost (see **Fig. 4.**)
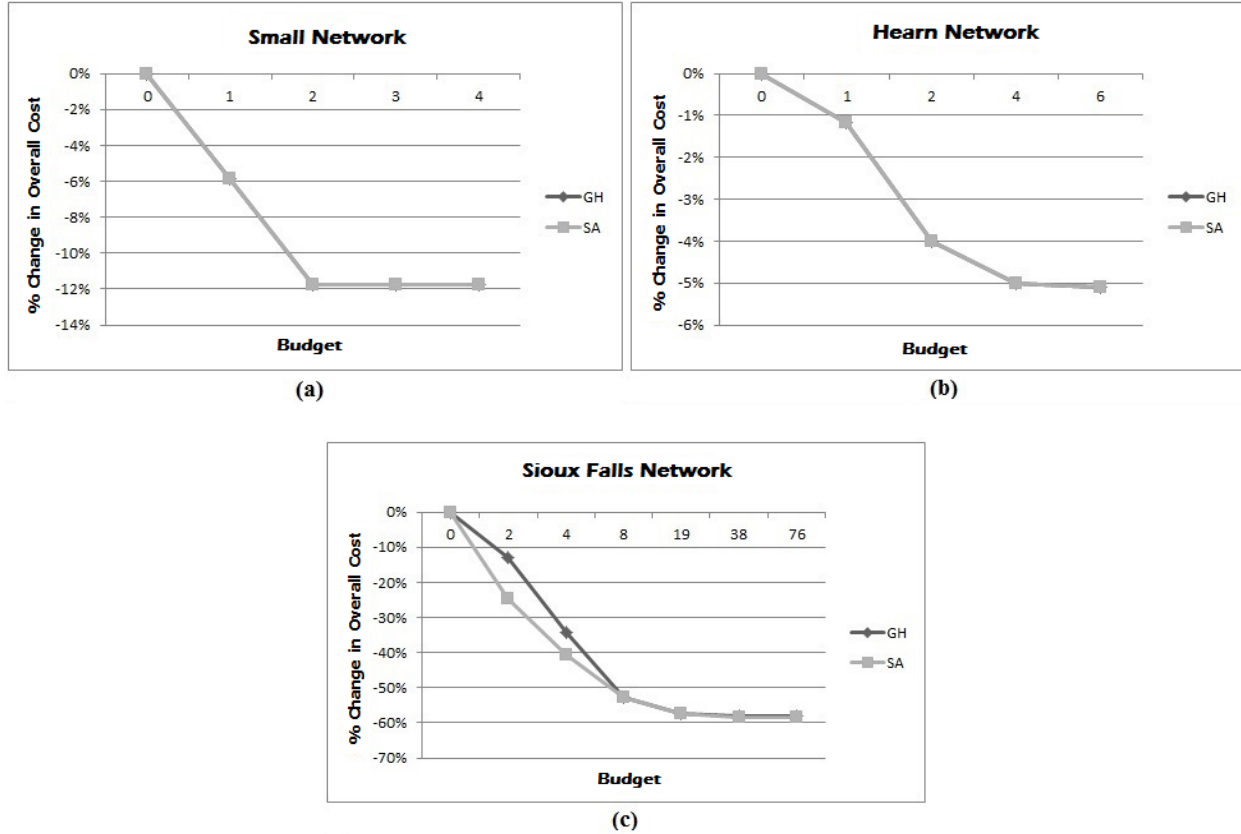
Fig.4. Percent change in the overall cost for different budget values:

(a) Small Network, (b) Hearn Network, and (c) Sioux Falls Network

The reason that such a point exists is that after a certain number of S&C are installed and separate walkways are available for pedestrians to use, the pedestrians no longer walk along the streets and interfere auto and transit's traffic. So, adding more S&C can no longer reduce the transportation cost. Therefore, it is not recommended to install S&C at every possible location (i.e. it is not the optimal design), even if we can afford the costs.

For the hypothetical Small and the Hearn networks, there is no significant difference between the quality of the solutions produced by GH and SA as shown in **Fig. 4 (a)** and **(b)**; however, for the much larger Sioux Falls network, the SA produces better solutions and outperforms the GH for low budget (**Fig. 4 (c)**). As the budget increases, the difference between the two algorithms diminishes. However, SA is still faster (**Fig. 3 (c)**).

Although optimally installing S&C decreases the overall cost in transportation networks, it has different impacts on the three modes of transportation. For auto and transit modes, installing

S&C means building separate walkways for pedestrians, and it can decrease the travel time of auto and transit modes. However, for pedestrians, installing separate walkways can decrease crashes and therefore increase safety. For example in the Sioux Falls network, as shown in **Fig. 5,** installing S&C causes larger reduction over walking travel cost than transit and auto cost. This can be explained by the effect that S&C have on pedestrians' safety (equation (4)).
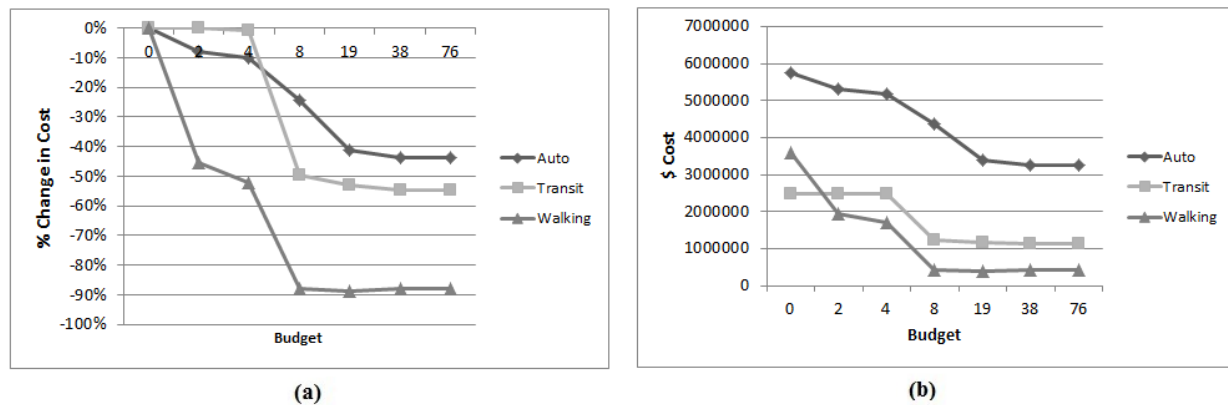

(a)   (b)

Fig.5. Changes in the overall transportation cost over different transportation mode for the Sioux Falls network: (a) Relative changes (percent of changes) in the overall cost, (b) Absolute changes in the overall cost.

4.3. Sensitivity analysis: Demand

To see the impact of installing S&C on a transportation network under different demand, we did a sensitivity analysis (shown in **Fig. 6**). A multiplier was used for demand, called "demand factor". Using the demand factor, we changed the demand in the networks from 0.25% of the default value up to 800%.

The results show that when the networks are less crowded (i.e. low traffic conditions), installing S&C do not have significant impact on the overall transportation cost. As the demand increases and more people use the network, S&C start to have more impact on the network until a certain point, where the network becomes overcrowded and the impact of S&C diminishes (**Fig. 6 (a)** and **(b)**) or remains unchanged (**Fig. 6 (c)**).
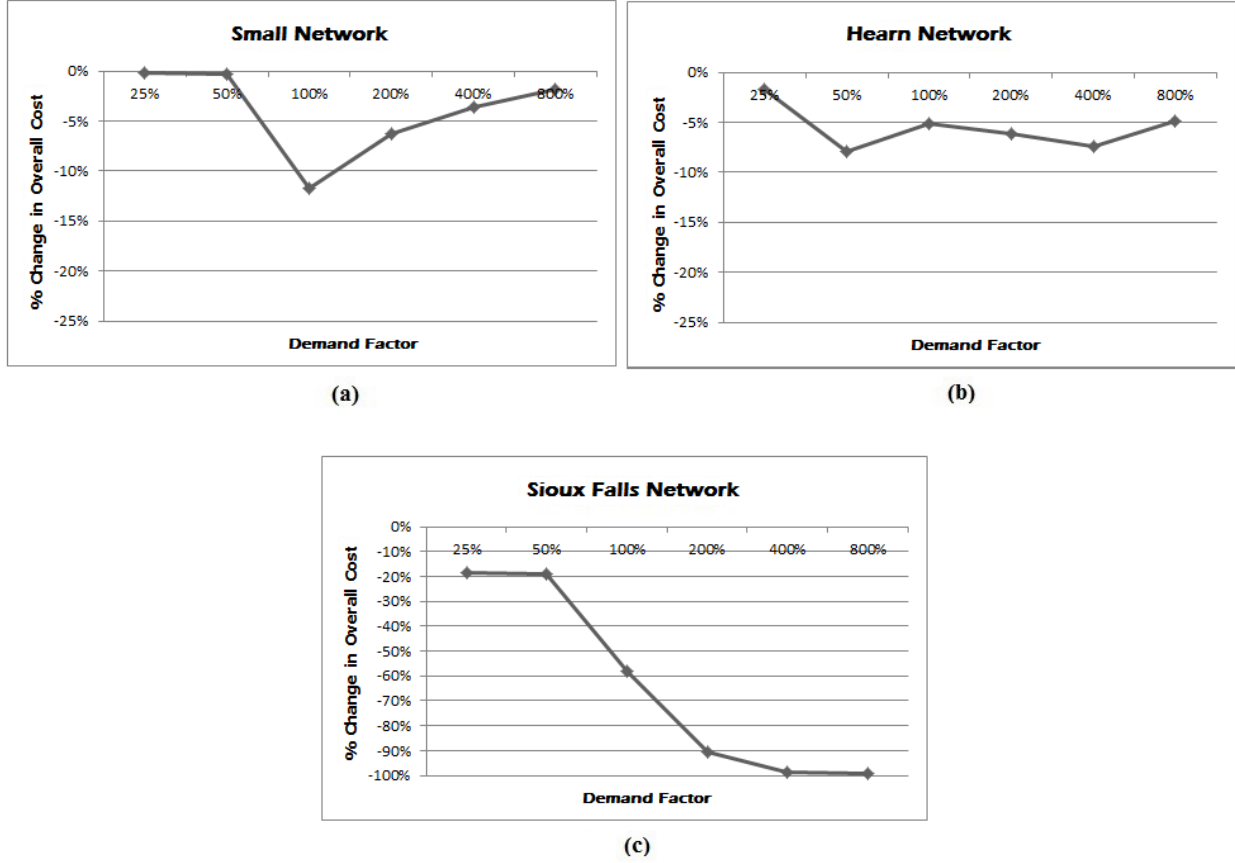
Fig. 6. Percent change in the overall cost due to traffic calming facilities for different demand factor for each OD pair: (a) Small network, (b) Hearn network, and (c) Sioux Falls network

4.4. Sensitivity analysis: Safety weight factor

In the travel cost function for sidewalk links (equation (4)) we used a safety weight factor, δ, to tradeoff travel cost and safety. This safety weight factor is used to combine the travel time part and the safety part in the travel cost function of sidewalk links. To investigate the impact of safety on the overall cost in the networks, and to find a compromise between safety and time in the objective function, we performed another set of experiments by changing the safety weight factor's value ranging from 0.0 to 1. As shown in **Fig. 7.**, for all values of the safety weight factor, installing S&C decreases the overall cost in all three networks. As the safety weight factors value increases, so does the impact of S&C on the overall cost (the overall transportation cost decreases more). This impact is more significant in the Sioux Falls network (see **Fig. 7.**)
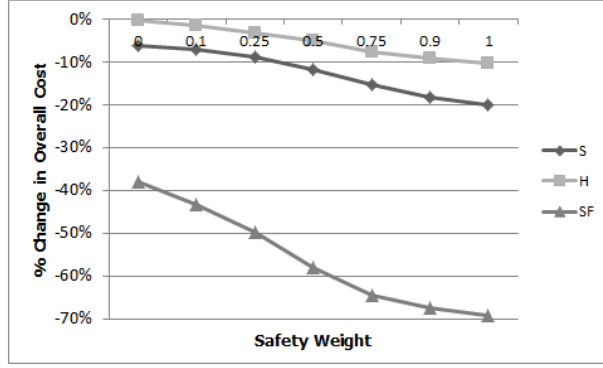
Fig.7. Percent change in the overall cost considering different values for the safety weight factor (S = Small Network, H = Hearn Network, SF = Sioux Falls Network)

4.5. *Flow demonstration in the Small Network*

To see the impact of installing S&C on the traffic flow in transportation networks considering changes in demand and safety weight factor, we conducted a series of experiments on the Small network. We investigated the travelers' flows in the network under three scenarios: (1) before and after installing S&C (the safety weight factor is $\delta = 0.5$, demand factor = 100%), (2) for low and high traffic congestion after installing S&C (for low and high demand factor), and (3) for low and high values of safety weight factor (after installing S&C).

Figs. 8.1.a. and **Fig. 8.1.b.** respectively show the flows in the network before and after installing S&C. In general, we observed that after installing S&C, less auto and more walking is used. More specifically, the auto link (1, 3) is no longer used after installing S&C, and the unused transit link (1001, 3001) is used after installing S&C. Some walking links are also used more often after installing S&C, such as (2, 324), (324, 431), and (431, 413). The transit link (3001, 4001) is used less after installing S&C.

Figures **8.2.a** and **8.2.b**. show the difference for the flow in the Small network after installing S&C under low and high traffic congestion (low and high demand factor) respectively. The model suggests that when the transportation network becomes more crowded, less auto and more walking and transit are used. This result is expected: when there is high auto traffic congestion; people are more likely to walk than to use their own vehicles because auto traffic congestion causes slow travel speeds.
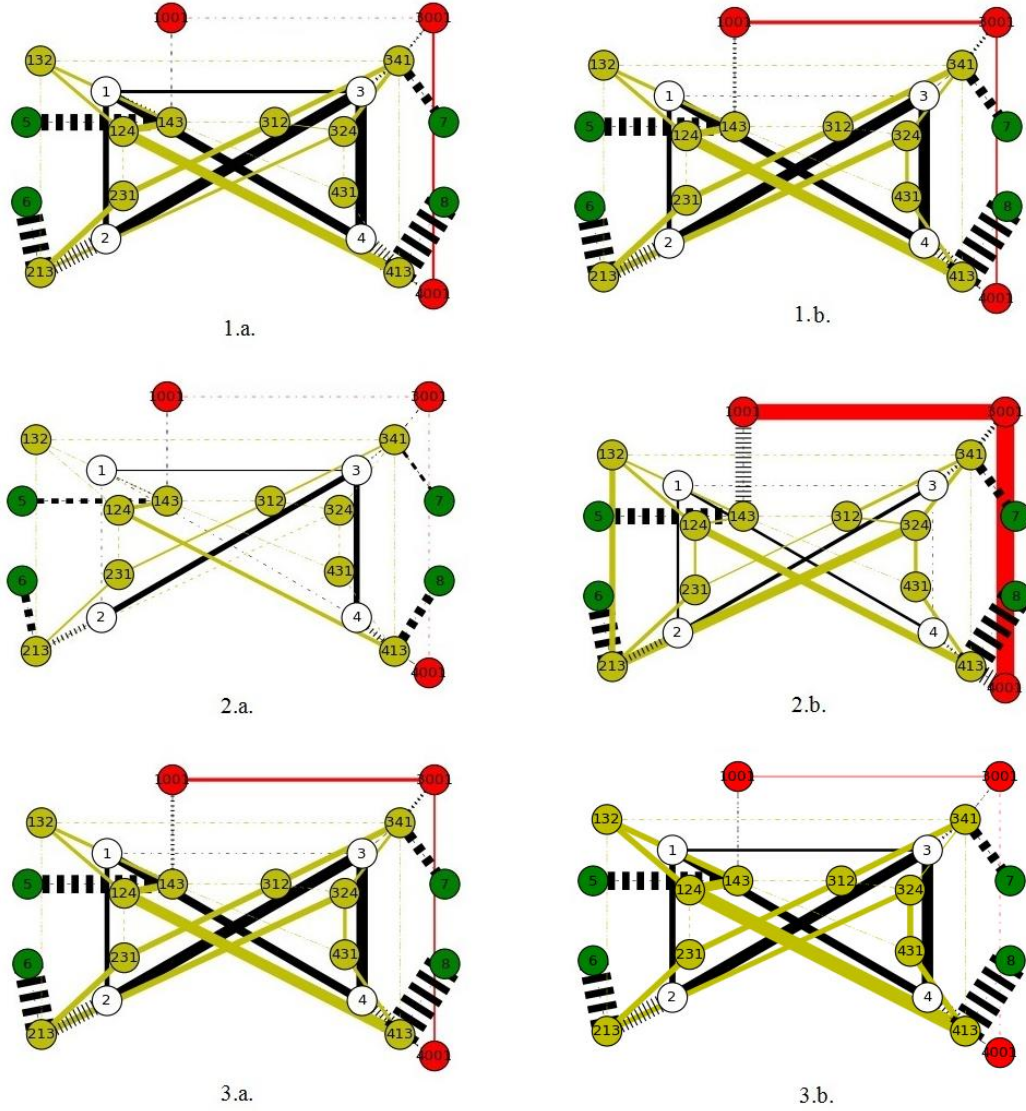
Fig.8. The flow in the Small Transportation Network under different scenarios: 1. before (a) and after (b) implementing traffic calming facilities, 2. for low (a) and high (b) crowd congestion, and 3. for low (a) and high (b) safety weight factor.

Figures **8.3.a** and **8.3.b.** show the travelers' flow in the Small transportation network for low safety weight ($\delta = 0.1$) and high safety weight ($\delta = 0.9$). As shown in **Figs. 8.3.a** and **8.3.b**, the auto links (1, 3) and (2, 1) and the walking links (124, 413), (143, 124), (132, 124), (324, 431), and (431, 413) are used more often for higher value of safety weight factor. On the other hand, transit links (1001, 3001) and (3001, 4001) are used less often. We speculate that increasing the safety weight factor (and therefore putting more emphasis on pedestrians' safety) leads to more use of walking and auto links, and less use of the transit mode. The presence of safe walkways that are separate from roads can encourage more people to walk. As a result, the flow of

27

pedestrians would cause less disturbance on the flow of auto, which can decrease the travel time for auto, increase the use of auto and decrease that of transit.

5. Conclusion

This study provides insights into how implementing traffic calming facilities such as installing sidewalks and crosswalks (S&C) affects pedestrians' safety and the transportation cost in a multimodal transportation network. This study considers pedestrians' safety as an important factor in designing a transportation network. A mixed-integer nonlinear programming model is developed for optimally locating S&C in a transportation network. The model is implemented in YALMIP (version 20141030) and solved using the BARON solver (version v1.69), one of the most advanced solvers in the market. However, because of the computational difficulty for the YALMIP/BARON MINLP formulation, two heuristic methods, a customized Greedy Heuristic and a Simulated Annealing algorithm, are developed for the problem. Experiments with three sample networks show that these algorithms outperform the BARON solver. Specifically, the SA algorithm is more efficient in producing better quality solutions for the Sioux Falls network (the larger sample network). Although these two approximate algorithms do not necessarily produce optimal solutions (i.e. the optimal design of a transportation network), the results they produce can help in better understanding of the impact of traffic calming facilities (i.e. S&C in this study) in multimodal transportation networks. The results show that installing traffic calmings according to the solution obtained by the SA algorithm reduces the total transportation cost by 12%, 5% and 59% respectively for the Small, Hearn and Sioux Falls networks. The optimal solutions are expected to yield an even greater reduction in transportation cost.

The major contribution of this research is to develop a quantitative network design model for locating traffic calming facilities in the transportation network to reduce the overall cost and to improve pedestrians' safety. The results suggest that not only does installing S&C improve pedestrians' safety, it also reduces the total transportation cost (including the travel cost of auto and transit). Installing S&C can provide separate walkways for pedestrians; as a result, pedestrians no longer have to walk along busy streets and interfere auto and transit's traffic. This can reduce the number of pedestrians' crashes and improve pedestrians' safety. It also can decrease the effect of pedestrians' traffic on auto and transit modes, which in turn reduces the

travel cost for auto and transit. However, S&C provide a smaller reduction of the total transportation cost in a network with low traffic. The results also suggest that installing S&C as safe walkways for pedestrians can encourage walking. This effect is consistent with the results of previous studies (e.g. (Gallimore et al., 2011; Pucher and Dijkstra, 2003; Southworth, 2005; Staunton et al., 2003)). More walking and less use of cars can improve public health (Lee and Buchner, 2008), alleviate traffic congestion, cut energy use and carbon emission and reduce noise and air pollution (Marshall and Garrick, 2010; Pucher and Dijkstra, 2003; Southworth, 2005).

This study can be viewed as a foundation for further research on pedestrians' transportation. Future research can be conducted in several directions. First, the only traffic calming facilities considered in this paper are sidewalks and crosswalks. However, there are many more to consider such as speed bumps, stop lights, stop signs and police patrol. On the other hand, in this study we only considered the safety effects of sidewalks (through equation (4)); one can also study the effect of crosswalks on pedestrians' safety. We assumed that the cost for installing S&C at any location in a transportation network is the same. This assumption can be relaxed by acquiring relevant data. Although safety is an important factor in promoting walking, connectivity is also important in designing a walkable transportation system (Southworth, 2005). Restrictions can be added to ensure connectivity when designing walkways. The transportation modes we considered in this study are walking, auto, and transit. One can consider other transportation modes such as bicycles. Further, the problem we studied in this paper is deterministic. Therefore considering uncertainty (in demand, capacity, travel time etc.) is another way for extending this research. Regarding the solution methodology, since the heuristic methods used in this study produce approximate solutions, a useful next step is to develop exact methods. As the problem is bi-level in nature, implementing a decomposition based method is recommended.

Acknowledgement

Appendix

Proof of Proposition 1: We will show that the objective function of the lower level problem, $\varphi_{l,m}(.)$, is a combination of convex functions, and therefore convex. First we will show that the objective function for auto link, $\varphi_{l,a}(.)$, (equation (9)) is convex. If we ignore ϑ and μ_{la} in (9) as they are positive constant values, and denote $x_{lm} = \sum_{k \in K} X_{klm}$ for all links (l, m) , then we can rewrite (9) as:

$$\varphi_{l,a}(X, Y) = \frac{1}{(\gamma_{l,s})^{\beta_1}} \left(x_{l,s1}{}^{\beta_1} + x_{l,s2}{}^{\beta_1} \right) + \frac{1}{(\gamma_{l,c})^{\beta_2}} \left(x_{l,bx}{}^{\beta_2} + x_{l,ex}{}^{\beta_2} \right)$$

$$+ t_{la} \left(1 + \alpha_1 \left(\frac{x_{la} + \omega x_{l,t}}{\gamma_{la}} \right)^{\beta_3} \right) + \mu_{l,a}$$

(16)

As x^a is convex for $a \geq 1 \ or \ a < 0$, therefore for $\beta_1, \beta_2 \geq 1 \ or \ \beta_1, \beta_2 < 0$ the first line (parts related to S&C) of (16) is convex. For simplicity we assume $\sum_{k \in K} X_{kla} = x$ and $\sum_{k \in K} X_{k,l,t} = y$, then the Hessian matrix of the last part in equation (16) (the second line) is:

$$H = \frac{1}{\gamma_{la}{}^2} \begin{bmatrix} \beta_3(\beta_3-1)\alpha_1 t_{la}(\frac{x+\omega y}{\gamma_{la}})^{\beta_3-2} & \beta_3(\beta_3-1)\alpha_1 t_{la}\omega(\frac{x+\omega y}{\gamma_{la}})^{\beta_3-2} \\ \beta_3(\beta_3-1)\alpha_1 t_{la}\omega(\frac{x+\omega y}{\gamma_{la}})^{\beta_3-2} & \beta_3(\beta_3-1)\alpha_1 t_{la}\omega^2(\frac{x+\omega y}{\gamma_{la}})^{\beta_3-2} \end{bmatrix}$$

A $n \times n$ real symmetric matrix M is positive semi-definite if $z^T \times M \times z \geq 0$ for all non-zero vectors z with real entries. For an arbitrary non-zero vector $z = [a \quad b]$, if we show that $z^T \times H \times z \geq 0$, then H is positive semidefinite.

$z^T \times H \times z = [a\beta_3(\beta_3-1)\alpha_1 t_{la}\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2} + b\beta_3(\beta_3-1)\alpha_1 t_{la}\omega(\frac{x+\omega y}{\gamma_{la}})^{\beta_3-2};$

$a\beta_3(\beta_3-1)\alpha_1 t_{la}\omega\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2} + b\beta_3(\beta_3-1)\alpha_1 t_{la}\omega^2\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2}] \times \begin{bmatrix} a \\ b \end{bmatrix} =$

$a^2\beta_3(\beta_3-1)\alpha_1 t_{la}\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2} + ab\beta_3(\beta_3-1)\alpha_1 t_{la}\omega\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2} + ba\beta_3(\beta_3-1)\alpha_1 t_{la}\omega\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2} +$

$b^2\beta_3(\beta_3-1)\alpha_1 t_{la}\omega^2\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2}$

We need to show that the above expression is non-negative:

$$a^2 \beta_3(\beta_3-1)\alpha_1 t_{la}\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2} + 2ab\beta_3(\beta_3-1)\alpha_1 t_{la}\omega\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2} +$$

$$b^2\beta_3(\beta_3-1)\alpha_1 t_{la}\omega^2\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2}$$

$$= \beta_3(\beta_3-1)\alpha_1 t_{la}\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2}(a^2 + 2ab\omega + b^2\omega^2)$$

$$= \beta_3(\beta_3-1)\alpha_1 t_{la}\left(\frac{x+\omega y}{\gamma_{la}}\right)^{\beta_3-2}(a+b\omega)^2$$

Knowing that x, y, t_{la}, ω and γ_{la} are all positive, for $\beta_3 \geq 1$ or $\beta_3 < 0$, the Hessian will always be non-negative, and therefore $\varphi_{l,a}(.)$ (represented by equation (9)) in the objective function is convex. In a similar way we can show that equations (10) and (13) are also convex. For $\beta_5 \geq 1$ or $\beta_5 < 0$ and positive values of $\propto_3, \gamma_{lp}$ and σ, equation (11) is convex as long as $P_{la}(X_{ls})X_{ls}$ is convex. In our case, $P_{la}(X_{ls})$ is a linear expression with positive coefficients, and therefore $P_{la}(X_{ls})X_{ls}$ is convex. Equation (14) consists of positive constant parameters and equation (15) is zero. Thus the objective function in the lower level problem is a sum of convex expressions, which is convex. □

References

Aashtiani, 1979. The multi-modal traffic assignment problem (Doctoral dissertation). Massachusetts Institute of Technology.

Aliakbarian, N., Dehghanian, F., Salari, M., 2015. A bi-level programming model for protection of hierarchical facilities under imminent attacks. Comput. Oper. Res. 64, 210–224.

Bahari, N.I., Arshad, A.K., Yahya, Z., 2013. Assessing the pedestrians' perception of the sidewalk facilities based on pedestrian travel purpose, in: 2013 IEEE 9th International Colloquium on Signal Processing and Its Applications. pp. 27–32.

Ben-Ayed, O., Blair, C.E., 1990. Computational difficulties of bilevel linear programming. Oper. Res. 38, 556–560.

Bouleimen, K., Lecocq, H., 2003. A new efficient simulated annealing algorithm for the resource-constrained project scheduling problem and its multiple mode version. Eur. J. Oper. Res. 149, 268–281.

Brouer, B.D., Desaulniers, G., Pisinger, D., 2014. A matheuristic for the liner shipping network design problem. Transp. Res. Part E Logist. Transp. Rev. 72, 42–59.

Bunn, F., Collier, T., Frost, C., Ker, K., Roberts, I., Wentz, R., 2003. Traffic calming for the prevention of road traffic injuries: systematic review and meta-analysis. Inj. Prev. 9, 200–204.

Campbell, B.J., Zegeer, C. V, Huang, H.H., Cynecki, M.J., 2004. A Review of Pedestrian Safety Research in the

United States and Abroad.

Cipriani, E., Petrelli, M., Fusco, G., 2006. A multimodal transit network design procedure for urban areas. Adv. Transp. Stud. 10.

Dill, J., 2004. Measuring Network Connectivity for Bicycling and Walking. Transp. Res. board Annu. Meet. 11–15.

Drezner, Z., Salhi, S., 2002. Using hybrid metaheuristics for the one-way and two-way network design problem. Nav. Res. Logist. 49, 449–463.

Drezner, Z., Salhi, S., Litman, T., 1999. Traffic calming: benefits, costs and equity impacts, Naval Research Logistics. Victoria Transport Policy Institute Victoria, BC,, Canada.

Elvik, R., 2001. Area-wide urban traffic calming schemes: a meta-analysis of safety effects. Accid. Anal. Prev. 33, 327–336. doi:10.1016/S0001-4575(00)00046-4

Fan, W., Machemehl, R.B., 2008. Tabu search strategies for the public transportation network optimizations with variable transit demand. Comput. Civ. Infrastruct. Eng. 23, 502–520.

Fan, W., Machemehl, R.B., 2006a. Optimal transit route network design problem with variable transit demand: genetic algorithm approach. J. Transp. Eng. 132, 40–51.

Fan, W., Machemehl, R.B., 2006b. Using a simulated annealing algorithm to solve the transit route network design problem. J. Transp. Eng. 132, 122–132.

Farahani, R.Z., Miandoabchi, E., Szeto, W.Y., Rashidi, H., 2013. A review of urban transportation network design problems. Eur. J. Oper. Res. 229, 281–302.

Farvaresh, H., Sepehri, M.M., 2011. A single-level mixed integer linear formulation for a bi-level discrete network design problem. Transp. Res. Part E Logist. Transp. Rev. 47, 623–640.

FHWA, 2010. Safety Benefits of Walkways, Sidewalks, and Paved Shoulders. Technical report, U.S. Dept. of Transportation, Federal Highway Administration, Office of Safety, Washington, D.C. [WWW Document]. URL http://safety.fhwa.dot.gov/ped_bike/tools_solve/walkways_trifold/ (accessed 4.22.14).

FHWA, ITE, 1999. Traffic Calming: State of the Practice. Report by Institute of Transportation Engineers [WWW Document]. Inst. Transp. Eng. URL http://www.ite.org/traffic/tcstate.asp

Frank, M., Wolfe, P., 1956. An algorithm for quadratic programming. Nav. Res. Logist. Q. 3, 95–110.

Fruin, J., 1992. Designing for pedestrians. Public Transp. United States.

Gallimore, J.M., Brown, B.B., Werner, C.M., 2011. Walking routes to school in new urban and suburban neighborhoods: An environmental walkability analysis of blocks and routes. J. Environ. Psychol. 31, 184–191.

Gallo, M., Montella, B., D'Acierno, L., 2011. The transit network design problem with elastic demand and internalisation of external costs: An application to rail frequency optimisation. Transp. Res. Part C Emerg. Technol. 19, 1276–1305.

Gårder, P.E., 2004. The impact of speed and other variables on pedestrian safety in Maine. Accid. Anal. Prev. 36, 533–542.

Hearn, D.W., Ramana, M. V., 1998. Solving congestion toll pricing models. Springer US.

Huang, H.F., Cynecki, M.J., 2000. Effects of traffic calming measures on pedestrian and motorist behavior. Transp. Res. Rec. J. Transp. Res. Board 1705, 26–31.

Kia, R., Baboli, A., Javadian, N., Tavakkoli-Moghaddam, R., Kazemi, M., Khorrami, J., 2012. Solving a group layout design model of a dynamic cellular manufacturing system with alternative process routings, lot splitting and flexible reconfiguration by simulated annealing. Comput. Oper. Res. 39, 2642–2658.

Kirkpatrick, S., Vecchi, M.P., 1983. Optimization by simulated annealing. Science (80-.). 220, 671–680.

LeBlanc, L.J., 1975. An algorithm for the discrete network design problem. Transp. Sci. 9, 183–199.

Lee, G., Joo, S., Oh, C., Choi, K., 2013. An evaluation framework for traffic calming measures in residential areas. Transp. Res. Part D Transp. Environ. 25, 68–76.

Lee, I.-M., Buchner, D.M., 2008. The importance of walking to public health. Med. Sci. Sports Exerc. 40, S512–8.

Lee, Y.-J., Vuchic, V.R., 2005. Transit network design with variable demand. J. Transp. Eng. 131, 1–10.

Loeb, P.D., Clarke, W.A., 2009. The cell phone effect on pedestrian fatalities. Transp. Res. Part E Logist. Transp. Rev. 45, 284–290.

Lofberg, J., 2004. YALMIP: A toolbox for modeling and optimization in MATLAB, in: Computer Aided Control Systems Design, 2004 IEEE International Symposium on. pp. 284–289.

Madadi, A., Kurz, M.E., Mason, S.J., Taaffe, K.M., 2014. Supply chain design under quality disruptions and tainted materials delivery. Transp. Res. Part E Logist. Transp. Rev. 67, 105–123.

Marshall, W.E., Garrick, N.W., 2010. Effect of street network design on walking and biking. Transp. Res. Rec. J. Transp. Res. Board 2198, 103–115.

Mesbah, M., Sarvi, M., Currie, G., 2008. New methodology for optimizing transit priority at the network level. Transp. Res. Rec. J. Transp. Res. Board 2089, 93–100.

Miandoabchi, E., Daneshzand, F., Szeto, W.Y., Farahani, R.Z., 2013. Multi-objective discrete urban road network design. Comput. Oper. Res. 40, 2429–2449.

Miandoabchi, E., Farahani, R.Z., Dullaert, W., Szeto, W.Y., 2011a. Hybrid Evolutionary Metaheuristics for Concurrent Multi-Objective Design of Urban Road and Public Transit Networks. Networks Spat. Econ. 12, 441–480.

Miandoabchi, E., Farahani, R.Z., Szeto, W.Y., 2011b. Bi-objective bimodal urban road network design using hybrid metaheuristics. Cent. Eur. J. Oper. Res. 20, 583–621.

Mitchell, D.H., MacGregor Smith, J., 2001. Topological network design of pedestrian networks. Transp. Res. Part B Methodol. 35, 107–135.

Parsafard, M., Rashidi, E., Li, X., Medal, H., 2015. A Multimodal User Equilibrium Traffic Assignment with Network Reconstruction.

Pucher, J., Dijkstra, L., 2003. Promoting safe walking and cycling to improve public health: lessons from the Netherlands and Germany. Am. J. Public Health 93, 1509–1516.

Randall, T.A., Baetz, B.W., 2001. Evaluating pedestrian connectivity for suburban sustainability. J. Urban Plan. Dev. 127, 1–15.

Resat, H.G., Turkay, M., 2015. Design and operation of intermodal transportation network in the Marmara region of Turkey. Transp. Res. Part E Logist. Transp. Rev. 83, 16–33.

Retting, R.A., Ferguson, S.A., McCartt, A.T., 2003. A review of evidence-based traffic engineering measures

designed to reduce pedestrian-motor vehicle crashes. Am. J. Public Health 93, 1456–1463.

Sahinidis, N. ~V., 2014. BARON 14.3.1: Global Optimization of Mixed-Integer Nonlinear Programs, User's Manual.

Seneviratne, P.N., Morrall, J.F., 1985. Analysis of factors affecting the choice of route of pedestrians. Transp. Plan. Technol. 10, 147–159.

Smith, J., 2001. Stochastic Modeling and Design of Vehicular and Pedestrian Networks. Transp. Res. Rec. J. Transp. Res. Board 1776, 211–219.

Southworth, M., 2005. Designing the walkable city. J. urban Plan. Dev. 131, 246–257.

Staunton, C.E., Hubsmith, D., Kallins, W., 2003. Promoting safe walking and biking to school: the Marin County success story. Am. J. Public Health 93, 1431–1434.

Toobaie, S., Aashtiani, H.Z., Hamedi, M., Haghani, A., 2010. Application of a complementarity approach to large scale traffic equilibrium problems, in: Transportation Research Board. pp. 1–29.

Vandervalk, A., Louch, H., Guerre, J., Margiotta, R., 2014. Incorporating Reliability Performance Measures Into the Transportation Planning and Programming Processes: Technical Reference.

Weinstein Agrawal, A., Schlossberg, M., Irvin, K., 2008. How Far, by Which Route and Why? A Spatial Analysis of Pedestrian Preference. J. Urban Des. 13, 81–98.

Yamada, T., Febri, Z., 2015. Freight transport network design using particle swarm optimisation in supply chain-- transport supernetwork equilibrium. Transp. Res. Part E Logist. Transp. Rev. 75, 164–187.

Yang, Z., Yu, B., Cheng, C., 2007. A Parallel Ant Colony Algorithm for Bus Network Optimization. Comput. Civ. Infrastruct. Eng. 22, 44–55.