# Vulnerability Assessment and Re-Routing of Freight Trains under Disruptions: A Coal Supply Chain Network Application

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## 1. Introduction

Today, our society depends on its transportation systems more than ever. A large percentage of the products we consume are transported long distances by road, rail, air, or a combination of modes. In addition, many people travel on roads to go to work every day. One of the main risks embedded in transportation systems is the failure of infrastructure elements such as bridges, tunnels, and facilities. These elements can fail due to natural disasters, terrorist attacks, or just because they are in bad condition. The failure of these elements can cause several different impacts including loss of life, economic loss, increased travel costs and congestion since the routes need to be changed to avoid the failed elements.

Rail transportation is an important and growing component of freight transportation in the United States. The benefits of rail transportation are that it is cheaper and produces less carbon emissions than road transportation. It is also easier to transport heavy loads on rail than on truck. Leaders in transportation are trying to increase the volume of goods transported by rail to alleviate loads on the road network and reduce carbon emissions. Large freight companies also are moving more of their transportation to a combination of rail and road.

There are several aspects of rail transportation that make it different than other transportation modes. First, the operations of a railroad are more centrally controlled than in the road network. That is, train operators have less autonomy to choose their own routes and schedules. Second, compared to road transportation, there is not as much excess capacity in rail transportation. Thus, it is important to consider capacity when routing and scheduling.

Several events in the last 30 years illustrate that the freight rail transportation system in the United States is vulnerable to disruptions. In 1993, flooding of the Mississippi and Missouri rivers caused several railroads to experience delays and cancellations. The estimated total cost of the disruption was \$ 182 million [12]. In 1996, a merger between Union Pacific and Southern Pacific railroads led to delays for many of Union Pacific's customers [26]. In 2005, a derailment on a main line in Wyoming near the Powder River Basin led to a shortage of coal in many parts of the United States as well as price increases [5]. Finally, after the death of Osama Bin Laden, it was revealed that Al-Qaeda was planning an attack on the rail infrastructure in the United States [6].

Disruptions have a large impact on rail transportation because there are less alternate routes available when a disruption occurs. There are several reason for the lack of alternate routes. First, rail is not as ubiquitous as roads. Second, much of the track in the United States is single line track. Thus, only one train can be on the track at a time in either direction. This makes it more difficult to reroute trains after a disruption. Third, the operation of a railyard can be complex and therefore it is difficult for a railyard to accommodate a lot of excess capacity. Again, this must be taken into consideration when rerouting.

Given the high vulnerability of the rail transportation, minimizing the risk of potential disruptions while transporting goods by rail is a challenging task. In this paper, we present a mathematical model for estimating the consequence of a disruption to a transportation network in which an attacker optimally chooses a set of infrastructure elements to attack in order to maximize the total disruption to the network. In addition to modeling the threat of an attacker, this model can also be used to determine critical elements of the network by identifying the set of elements of the rail network whose unavailability causes the largest consequence. The consequence estimation of attacker's decisions are also taken into account in a unit train transportation system by modeling trains as discrete demand units that stay intact from origin to destination. The model captures the movement of trains in time and space over a finite time horizon. Tracks and railyards in the network have strict capacity constraints for short time periods.

In this paper, we consider rail transportation of bulk commodities such as coal, grain, and scrap metal since they make up a large percentage of the volume transported on rail. In bulk transportation, demand is in terms of entire trains; therefore, there is no need to switch cars at intermediate classification yards.

The demand for commodities is also smoother than the demand for lower volume items. For example, several power plants in the southern United States place a fixed-quantity order every month.Coal combustion has been commonly used to generate electricity and provide power for many kinds of operations in the United States. In 2008, it was announced that 48.2% of the electricity consumed in the US was produced by the combustion of coal in coal power plants [32]. The electricity generated in these plants is being used in many areas such as: hospital operations, vaccine storage, security and surveillance systems, as well as water treatment. Hence, in order to keep this source of electricity safe for such important services in case of a disruption or disaster, operations in the coal supply chain must be secured. Moreover, coal transportation is a good representative of bulk transportation by rail, and therefore a good source of data to test the proposed model, since 70% of coal was transported by rail throughout the U.S. in 2010 [33].

After the coal is mined, it is sent to a processing facility where the coal pieces are crushed into more manageable chunks. The trains typically consist of 125 to 150 cars loaded with between 110-120 tons of coal in each rail car. These trains are dispatched on their routes towards specific power plants. Even though the primary objective in the coal supply chain is to meet electricity demand, reducing the transportation and storage costs of coal as much as possible is also a major consideration.

Most power plants are designed in such a way that they can only use a single type of coal in order to generate electricity. Hence, there could be serious results of a disruption or a disaster that occurs in the coal supply chain, especially for the areas of the country that rely heavily on electricity generated from the coal mined in Wyoming's Powder River Basin (PRB). In this case study, the sub-bituminous coal transportation network is used. Sub-bituminous is the most common type of coal mined in PRB. The PRB accounts for about 40 % of all consumption within the US [33]. This particular coal type has significantly low $SO_2$ emissions and cannot produce high energy output. However, many energy companies are automatically attracted by the low emissions level and abundance of supply of this type of coal.

While transporting coal from mines to power plants, many important constraints are observed in the coal supply chain. The amount of coal that can be carried by a train is restricted by the size of the trains used in the system. Also, depending on the sizes of these trains, some trains can only travel on special tracks. The availability of coal in different time periods for loading/unloading operations requires extra planning. Hence, there could be serious results of a disruption or a disaster that occurs in the coal supply chain, especially for the areas of the country that rely heavily on electricity generated from the coal mined in the PRB.

The remainder of the paper is organized as follows. Section 2 highlights the most relevant studies in the literature focusing on disruptions in rail transportation networks. A formal problem description, the proposed two-stage mathematical model formulation and our solution methodology are presented in Section 3. Section 4 demonstrates the analysis of the computational results obtained by using a real coal supply chain network. Finally, conclusions and future work are described in Section 5 .

## 2. Literature review

There is an established body of research on modeling the rail transportation system [3, 2, 21]. Crainic [10] surveys the research on freight transportation. He discusses three planning levels:

strategic, tactical, and operational. In the strategic level, long-term decisions are made such as where to locate yards and where to build rail lines. At the tactical level, medium-term decisions are made such as the routing of trains and aggregate scheduling. The operational level includes shorter-term decisions such as crew scheduling and locomotive scheduling.

There are two types of rail transportation: merchandise trains and unit trains. Merchandise trains are composed of cars with different destinations. Therefore, consolidation, or blocking, is a crucial part of merchandise train operations. Partly due to the challenging problems associated with the blocking process, most of the research on rail transportation from an operations research perspective has considered merchandise trains (see Nemani and Ahuja [21]). Unit trains are composed of cars with the same destination; thus, blocking is no longer needed. There is not as much research on unit train transportation. Lawley et al. [18] present a time-space routing and scheduling model for unit trains. Their model accounts for both loaded and empty trains. The second stage of our interdiction model is similar to this model except that we do not account for empty trains. Several authors have developed approaches to assess railway capacity [17, 7, 20, 1]. We assume that railyards and tracks have discrete unit train capacity for each time period in the planning horizon.

Because of the prevalence of disruptions in transportation networks, there has been a significant amount of work on managing the recovery from a disruption. Applications include machine scheduling [25], production-inventory systems [35], supply chains [24], passenger air transportation [16], passenger rail transportation [15], and project scheduling [37].

One way to study the vulnerability of a network is to identify the critical nodes and edges of the network. Interdiction models identify critical nodes and edges by modeling a game between an adversary and the operator of the network, who routes flow through the network after the adversary makes his attack. Fulkerson and Harding [11] are among the first to study how to interdict arcs in a network to maximally increase the length of the shortest path; they are followed by others [13]. Wollmer [34] is among the first to provide a model for interdicting a maximum-flow network. Others have extended this problem to consider probabilistically successful attacks [9, 14] and multiple objectives [29, 28, 27]. Researchers have also considered other objectives such as minimizing the maximum reliability path [22] and minimizing the maximum profit [19]. Further, Church et al. [8] present models for interdicting a set of facilities.

Researchers have also begun to study the vulnerability of freight rail networks. Peterson and Church [23] describe models for the impact of a disruption to the United States freight transportation network. They present an uncapacitated model that is a modification of the shortest path problem. They also present a continuous multicommodity network flow model that has line capacities. Babick [4] models the allocation of security resources to the rail network in the state of California as a defender-attacker-operator problem, represented by a bi-level mixed-integer programming formulation. Both studies treat the rail transportation as a continuous static network flow problem. In our work, we model the rail transportation system as a discrete dynamic network flow problem.

As mentioned above, there have been many studies on how to reduce network risks that can be applied to transportation networks. However, the mathematical models employed in these studies do not have enough detail to be applied directly to rail networks where disruptions are allowed. For instance, existing models mostly model goods as continuous (divisible quantities). However, in most settings trains can only be realistically modeled using discrete units of flow. Moreover, current models are usually static, meaning that all flow happens at the same time. Although real flows are almost never static, modeling flows as static is appropriate for uncapacitated networks or networks in which there are capacity constraints over long time periods (e.g., a month) but there are not strict capacity constraints for shorter time intervals (e.g., day or hour). Even though static flow models are also appropriate for strategic level decisions, they do not provide enough resolution to identify the impacts of disruptions on rail networks with short term capacities on railyards and tracks.

Thus, this paper differs from previous studies in that it explicitly incorporates disruptions and discrete unit flows of trains on rail networks with discrete, varying capacities with time. The factors considered in our model result in realistic scenarios that lead to tactical and operational level vulnerability assessment analyses including rerouting decisions, travel and delay costs analysis, and the frequency of interdictions of facilities for a dynamic rail transportation system. Furthermore, decision makers can utilize the proposed web-based decision support tool to monitor the changes in the rail network.

## 3. Mathematical model development

In this section, we model the problem of identifying critical elements as a two-player game. In this game, an interdictor acts first and destroys a set of nodes and arcs. An operator follows the interdictor and chooses routes and schedules for trains given network elements that have not failed.

This game can be modeled as a bi-level integer program. Let $y$ be a vector of interdiction variables in which $y_i$ is 1 if node $i$ is destroyed and 0 otherwise. Let $f_i$ be the cost of interdicting node $i$. The interdictor has a budget of $b$ to spend on interdicting nodes. Let $Y$ be the feasible region of $y$ defined by the following constraints:

$$y_i \in \{0,1\} \quad \forall i \in \mathcal{N} \tag{1a}$$

$$\sum_{i \in \mathcal{N}} f_i y_i \leq b \tag{1b}$$

Table 1 introduces the sets, parameters and decision variables that are used in the two-stage integer-programming formulation of the interdiction model. In the following section, we introduce a bi-level interdiction model whose first stage is represented by the constraints (1a) and (1b).

### 3.1. Interdiction model

We propose a pure integer model for the second stage decisions after the disruption occurs. Routing and scheduling of trains is done given a network with available nodes and arcs after disruption. Therefore, our second stage IP model aims to satisfy the demands of plants with minimum cost and without eliminating the capacity restrictions of network elements while dispatching trains from mines to plants through predetermined routes. A time-indexed formulation captures the true capacity limitations of nodes and arcs in any given period. The flexibility of being able to arrange the length of the planning period provides great control on the scale of the problem as well. For the sake of simplicity, we only consider the flow of identical unit trains that carry the same amount of coal regardless of the origin destination pair they are assigned to. The model selects the cheapest route first and then schedules trains according to capacity and demand requirements.

Table 1: Notation

| | |
|---|---|
| **Sets** | |
| $N$ | set of all loading/unloading stations (nodes) |
| $M \subseteq N$ | set of mines |
| $P \subseteq N$ | set of plants |
| $A$ | set of all track segments |
| $R$ | set of feasible routes between all O–D pairs for all trains |
| $RT(r) \subseteq A$ | set of track segments included in the route of O–D pair $r \in R$ |
| $RN(r) \subseteq N$ | set of nodes included in the route of O–D pair $r \in R$ |
| $T$ | set of all time periods $\{\triangle, 2\triangle, \ldots, \|T\|\}$ |
| $D$ | set of days making up the planning horizon |
| $T(d)$ | set of time periods in day $d \in D$ |
| **Parameters** | |
| $\triangle \in Z^+$ | length of planning period |
| $K$ | number of planning periods in planning horizon |
| $o(r) \subseteq M$ | origin station of route $r \in R$ |
| $h(r) \subseteq P$ | destination station of route $r \in R$ (Plants) |
| $TC_{at}$ | track capacity of segment $a \in A$ at time $t \in T$ |
| $TC_{it}$ | track capacity of node $i \in N$ at time $t \in T$ |
| $\tau_r$ | total travel time of route $r \in R$ in multiples of $\triangle$ |
| $\tau_{ra}$ | travel time on route $r \in R$ to reach track segment $a \in RT(r)$ in multiples of $\triangle$ |
| $\tau_{ri}$ | travel time on route $r \in R$ to reach node $i \in RN(r)$ in multiples of $\triangle$ |
| $d_r$ | distance of route $r \in R$ |
| $c$ | cost per unit distance, which includes fixed costs (labor, equipment cost, etc.) and variable costs (fuel, maintenance cost, etc.) |
| $g_r$ | cost of moving one unit train on route $r \in R$ |
| $h_i$ | demand of station $i$ over the planning horizon |
| $l_t$ | length of the time period of $t \in T$ |
| $q$ | cost incurred when 1 train is delayed one time unit |
| $CR$ | cost of delay per time period / cost per unit distance = q/c |
| **Decision variables** | |
| $X_{rt}$ | number of trains departing from $o(r)$ on route $r \in R$ in time period $t \in T$ |
| $O_{rt}$ | number of trains waiting (or being loaded/unloaded) at $o(r)$ of route $r \in R$ in time period $t \in T$ |

Consider the following bi-level capacity-interdiction model:

$$\max_{y \in Y} \ell(y) = \quad \min \quad \sum_{r \in R} \sum_{t \in T} g_r X_{rt} + \sum_{r \in R} \sum_{t \in T} q l_t O_{rt} \tag{2a}$$

$$\text{s.t.} \quad O_{rt} + X_{rt} \geq O_{r,t-\triangle} \qquad \forall r \in \mathcal{R}, t = \triangle, \triangle + 1, \ldots, T \tag{2b}$$

$$\sum_{t \in T(d)} \sum_{r | i \in RN(r)} X_{r,t-\tau_{ri}} \leq TC_{id}(1 - y_i) \qquad \forall d \in \mathcal{D}, i \in \mathcal{N} \tag{2c}$$

$$\sum_{t \in T(d)} \sum_{r | a \in RT(r)} X_{r,t-\tau_{ra}} \leq TC_{ad} \qquad \forall d \in \mathcal{D}, a \in \mathcal{A} \tag{2d}$$

$$\sum_{t \in T} \sum_{r | i = h(r)} X_{rt} \geq h_i \qquad \forall i \in \mathcal{P} \tag{2e}$$

$$\sum_{r \in \mathcal{R}} (X_{rt} + O_{rt}) \leq n \quad \forall t \in \mathcal{T} \tag{2f}$$

$$X_{rt}, O_{rt} \in \mathbb{Z}^+ \qquad \forall r \in \mathcal{R}, t \in \mathcal{T} \tag{2g}$$

The objective function (2a) is the total cost incurred by 1) the total distance traveled and 2) the total delay incurred when trains have to wait at their origin stations. Constraints (2b) balance flow at the origin station of route $r$. For each planning period ($\triangle$) and route ($r$), the number of trains waiting at the origin node and departing the origin node to travel on route $r$ at time period $t$ must be greater than number of trains available at the origin node $\Delta$ time units before, at time period ($t - \triangle$). Constraints (2c) and (2d) guarantee that the flow of trains moving each day does not exceed the daily track segment and node capacity, respectively. Moreover, the right hand side of constraints (2c) forces an interdicted node to have zero capacity. Constraints (2e) state that the demand of each plant should be satisfied. Finally, constraints (2f) ensure that the number of trains waiting at the origin node and departing that node must be less than or equal to $n$, the number of trains, for each route and time period. The next section discusses the solution methodology developed to solve this version of the bi-level interdiction model.

### *3.2. Reformulation*

To solve problem (2), we consider solving a relaxation that involves reformulating the problem as a single level MIP. The relationship of this relaxation to the original formulation is discussed in the next section. The approach considered in this section fixes the $y$ variables and relaxes the integrality restriction on the $X$ and $O$ variables. Then, the dual of the inner minimization is taken.

Since both levels are then maximization after taking the inner dual, the bi-level problem reduces to a single level mixed-integer program.

First, relax the inner minimization problem:

$$
\begin{aligned}
\max_{y \in Y} \tilde{\ell}(y) = \quad \min \quad & \sum_{r \in R} \sum_{t \in T} g_r X_{rt} + \sum_{r \in R} \sum_{t \in T} q l_t O_{rt} \qquad \text{[duals]} && \text{(3a)} \\
\text{s.t.} \quad & O_{rt} + X_{rt} \geq O_{r,t-\triangle} \qquad \forall r \in \mathcal{R}, t = \triangle, \ldots, |T| \qquad [\alpha_{rt}] && \text{(3b)} \\
& \sum_{r | i \in \mathcal{N}(r)} \sum_{\substack{t \in \mathcal{T}(d) \\ t \geq \tau_{ri}}} X_{r,t-\tau_{ri}} \leq TC_{id}(1 - y_i) \qquad \forall i \in \mathcal{N}, d \in \mathcal{D} \qquad [\beta_{id}] && \text{(3c)} \\
& \sum_{r | a \in \mathcal{A}(r)} \sum_{\substack{t \in \mathcal{T}(d) \\ t \geq \tau_{ra}}} X_{r,t-\tau_{ra}} \leq TC_{ad} \qquad \forall a \in \mathcal{A}, d \in \mathcal{D} \qquad [\gamma_{ad}] && \text{(3d)} \\
& \sum_{t \in T} \sum_{r | i = h(r)} X_{rt} \geq h_i \qquad \forall i \in \mathcal{P} \qquad [\zeta_i] && \text{(3e)} \\
& \sum_{r \in \mathcal{R}} (X_{rt} + O_{rt}) \leq n \quad \forall t \in \mathcal{T} \qquad [\phi_t] && \text{(3f)} \\
& X_{rt}, O_{rt} \geq 0 \qquad \forall r \in \mathcal{R}, t \in \mathcal{T} \quad [\delta_{rt}, \eta_{rt}] && \text{(3g)}
\end{aligned}
$$

We now take the dual of the inner minimization. The resulting model is then:

$$
\begin{aligned}
\max_{y\in Y} \quad & \sum_{i\in\mathcal{N}}\sum_{d\in\mathcal{D}} TC_{id}(1-y_i)\beta_{id} + \sum_{a\in\mathcal{A}}\sum_{d\in\mathcal{D}} TC_{ad}\gamma_{ad} \\
& + \sum_{i\in\mathcal{P}} h_i\zeta_i + \sum_{t\in\mathcal{T}} n\phi_t && \text{(4a)} \\
\text{s.t.} \quad & \sum_{i\in\mathcal{N}(r)} \mathbb{I}\left\{\beta_{i,d(0+\tau_{ri})}\right\} + \sum_{a\in\mathcal{A}(r)} \mathbb{I}\left\{\gamma_{a,d(0+\tau_{ra})}\right\} \\
& +\zeta_{h(r)} + \phi_0 + \delta_{r0} \le g_r \quad \forall r\in\mathcal{R} && \text{(4b)} \\
& \alpha_{rt} + \sum_{i\in\mathcal{N}(r)} \mathbb{I}\left\{\beta_{i,d(t+\tau_{ri})}\right\} + \sum_{a\in\mathcal{A}(r)} \mathbb{I}\left\{\gamma_{a,d(t+\tau_{ra})}\right\} \\
& +\zeta_{h(r)} + \phi_t + \delta_{rt} \le g_r \quad \forall r\in\mathcal{R}, t=\triangle,\ldots,|T| && \text{(4c)} \\
& -\alpha_{r,1} + \phi_0 + \eta_{r0} \le ql_0 \quad \forall r\in\mathcal{R} && \text{(4d)} \\
& \alpha_{rt} - \alpha_{r,t+\Delta} + \phi_t + \eta_{rt} \le ql_t \quad \forall r\in\mathcal{R}, t=\triangle,\ldots,|T|-1 && \text{(4e)} \\
& \alpha_{r|T|} + \phi_{|T|} + \eta_{r|T|} \le ql_{|T|} \quad \forall r\in\mathcal{R} && \text{(4f)} \\
& \alpha_{rt} \le 0 \quad \forall r\in\mathcal{R}, t=\triangle,\ldots,|T| && \text{(4g)} \\
& \beta_{id} \le 0 \quad \forall i\in\mathcal{N}, d\in\mathcal{D} && \text{(4h)} \\
& \gamma_{ad} \le 0 \quad \forall d\in\mathcal{D}, a\in\mathcal{A} && \text{(4i)} \\
& \zeta_i \ge 0 \quad \forall i\in\mathcal{P} && \text{(4j)} \\
& \phi_t \le 0 \quad \forall t\in\mathcal{T} && \text{(4k)} \\
& \delta_{rt}, \eta_{rt} \ge 0 \quad \forall r\in\mathcal{R}, t\in\mathcal{T} && \text{(4l)}
\end{aligned}
$$

where $d(t)$ is the day of time period $t$, $|T|$ is the last time period, $\mathbb{I}\left\{\beta_{i,d(t+\tau_{ri})}\right\} = \beta_{i,d(t+\tau_{ri})}$ if $t+\tau_{ri} \le T$ and 0 otherwise, and $\mathbb{I}\left\{\gamma_{a,d(t+\tau_{ra})}\right\} = \gamma_{a,d(t+\tau_{ra})}$ if $t+\tau_{ra} \le T$ and 0 otherwise.

Notice that when we take the dual of the inner minimization problem, it changes the inner minimization problem to a maximization problem. Eliminating the maximization sign for the inner problem yields a single-level maximization model.

Also notice that our single-level model has now nonlinear terms $y_i\beta_{id}$. Since these nonlinear terms are a product of a binary variable and a continuous variable, we can linearize them by applying a technique described by Sherali and Alameddine [30]. First, substitute the non-negative variable $\kappa_{id} = y_i \beta_{id}$. Then, add the constraints:

$$\kappa_{id} - \underline{\beta}_{id} y_i \geq 0 \quad \forall i \in \mathcal{N}, d \in \mathcal{D} \tag{5a}$$

$$\kappa_{id} - \beta_{id} \geq 0 \quad \forall i \in \mathcal{N}, d \in \mathcal{D} \tag{5b}$$

with $\underline{\beta}_{id}$ denoting a lower bound of $\beta_{id}$.

This results in the following single-level MIP:

$$\begin{aligned} \max_{y \in Y} \quad & \sum_{i \in \mathcal{N}} \sum_{d \in \mathcal{D}} (TC_{id}\beta_{id} - TC_{id}\kappa_{id}) + \sum_{a \in \mathcal{A}} \sum_{d \in \mathcal{D}} TC_{ad}\gamma_{ad} \\ & + \sum_{i \in \mathcal{P}} h_i \zeta_i + \sum_{t \in \mathcal{T}} n\phi_t \qquad (6a) \\ \text{s.t.} \quad & \text{(4b)–(4l)} \\ & \text{(5a)–(5b).} \end{aligned}$$

## 4. Computational results

Note that we first relaxed the $X$ and $O$ decision variables of the second stage problem and then took the dual of it to obtain a MIP formulation. In practice, it is possible that the solution of the MIP can have fractional decision variables. However, our analysis shows that for our problems the proportion of fractional $X$ and $O$ variables is observed to be no greater than 0.2% and 1%, respectively.

In the following subsections, we first illustrate the basic properties of the networks used in model (6). Then, we demonstrate how these different networks impact the interdictor and attacker decisions. Our two-stage interdiction program is solved by using IBM ILOG CPLEX 12.1 on a single node (with two Intel six-core Xeon X5670 2.93 GHz processors and 24GB of memory) of a supercomputer.

### *4.1. Network and data construction process*

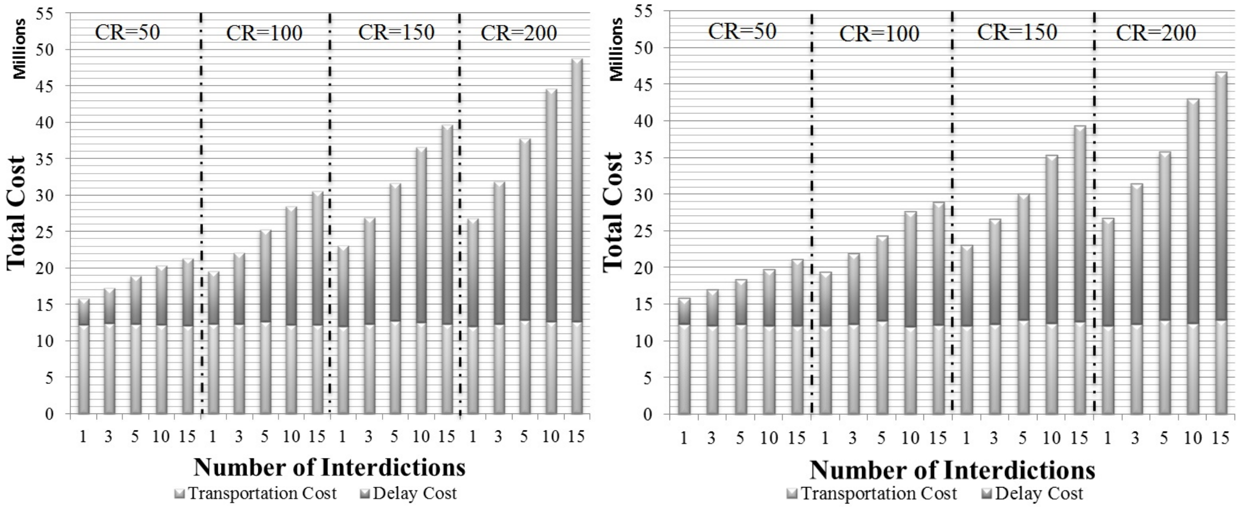
The coal plants and coal mines used to construct the real coal network are retrieved from the United States Coal Activity Map project [31]. Through focusing on large suppliers and consumers of coal, the refined network provides an initial representative model of the entire coal supply chain. The average production rate for each mine is approximately 2,000 tons, while the average consumption rate over all of the plants is 1,400 tons. There were a great number of mines and plants to consider based upon these averages. Therefore, the network is narrowed down by using the constraint that the average production and consumption rate at each mine or plant has to be greater than 5,000 tons. This value is chosen as a threshold for network reduction. Accordingly, there were 39 mines and 49 plants that fit the constraint. Then the mines and plants are found in which sub-bituminous coal is being mined and burned, respectively. The demand of the plants that burns sub-bituminous coal is estimated to be 57% of the total demand for coal in all plants. Also, the production at the sub-bituminous mines account for 76 % of the total coal production across 39 mines.

Based on the rail network data obtained through the steps described above, we labeled 8 sub-bituminous coal mines, 23 sub-bituminous coal plants and 37 rail yards as primary nodes. While generating routes between each mine-plant pair, we observe that there are 135,655 nodes (mines, plants, rail yards, tunnels, bridges and other connection points) and 172,888 arcs in the network. Hence, in order to generate a connected manageable rail network based on this dataset, we developed a trimming algorithm (see Algorithm Appendix A.1 in Section Appendix A.1 ) which explores K-shortest paths [36] between any combinations of mines and plants with $K = 3$ and $K = 10$ to obtain different sets of routes with sizes $R = 552$ and $R = 1840$, respectively. By using this procedure, we produced a connected graph with 456 nodes which includes 8 mines, 23 plants, 37 yards, 388 critical elements (tunnels, bridges, etc.) and 36,935 arcs. Using this reduced network, multiple routes (shortest paths) between coal plants and mines are generated to be used as input routes for the model (6).

### *4.2. Impacts of network size and budget level*

In this section, we assess the impacts of an interdiction budget ($b$), network size ($R$) and ratio of delay cost per time unit to the operation cost per unit distance ($CR$).

Note that there are three main costs included in the objective function of model 6. These cost items are total transportation (operation), total delay, and total interdiction costs. For the sake of simplicity, regardless of which node is interdicted, it is counted as one interdiction ($f_i = 1$). Hence, the total cost incurred by transporting coal from mines to plants and the total cost due to delays are the two main costs that the defender wants to minimize, while the attacker wants to maximize them.




(a) $R$=552 (b) $R$=1840

Figure 1: Cost components v.s. number of interdictions

Figures 1a and 1b demonstrate total transportation and delay costs on four different networks where the total number of routes in the network ($R$) is 552 and 1840, respectively. Changes in these cost terms are also observed by running model (6) for different cost ratios ($CR$) and attacker's budget levels ($b$). It is commonly seen in Figure 1 that for each $CR$ level, total transportation costs do not change significantly as the number of interdicted nodes in the network is increased. *However, delay costs increase dramatically compared to the total transportation cost as more nodes are interdicted.* This implies that as the interdictor manages to disable more routes and nodes in the network, it yields extra delays but train transportation can be handled at similar cost levels in each scenario. Similarly, total transportation cost remains almost at the same level despite varying $CR$ and $b$ parameters. One expects to see similar transportation costs with different $CR$ values since $CR$ is changed only by varying the cost of delaying a train for an hour not the cost of operating a route. However, based on the cost terms in Figure 1, we can see that neither higher budget levels nor larger $CR$ values resulted in a significant increase of total transportation costs in different networks. Model (6) finds similar minimum transportation costs regardless of which/how many nodes are interdicted for different $CR$ values. The main reason behind this phenomenon is that if an element on a route is interdicted, other alternative routes are made available with similar costs by the K-shortest path algorithm. Even though the cost of operating trains on several routes remains nearly the same, disruptions cause significant increases in delay costs as $b$ and $CR$ increase.

Table 2: Interdicted nodes with varying $CR$ when R=552

| b | CR=50 | CR=100 |
|---|---|---|
| 1 | 74 | 74 |
| 3 | 72-74-80 | 73-81-187 |
| 5 | 70-72-74-79-81 | 70-72-74-80-187 |
| 10 | 69-70-72-74-78-80-86-88-91-187 | 68-70-72-74-78-80-86-88-103-187 |
| 15 | 51-69-70-72-74-78-80-82-84-86-88-91-93-103-187 | 51-68-70-72-74-78-80-83-84-86-88-91-93-102-187 |

| b | CR=150 | CR=200 |
|---|---|---|
| 1 | 80 | 80 |
| 3 | 72-80-187 | 72-81-187 |
| 5 | 70-72-74-80-187 | 70-72-74-81-187 |
| 10 | 68-70-72-74-78-80-86-88-103-187 | 68-70-72-74-78-80-86-88-103-187 |
| 15 | 51-68-70-72-74-78-80-83-84-86-88-91-92-103-187 | 51-68-70-72-74-78-80-83-84-86-89-91-92-103-187 |

Tables 2 and 3 list the nodes that are interdicted in the scenarios provided in Figure 1. Most of the nodes interdicted with smaller budget levels are also attacked when the budget levels are increased. For instance, in Table 2, nodes 72, 74, and 80 are attacked when $CR = 50$ and $b = 3$. These three nodes are also taken out from the network when $CR = 50$ and $b = 10$, or 15. For the scenario with $b = 5$, two of these nodes (72, 74) are disabled by the attacker. Similarly, the frequency of interdicting a single or combinations of nodes in different networks with different values of $b$ and $CR$ can be reported.

Table 3: Interdicted nodes with varying $CR$ when R=1840

| b | CR=50 | CR=100 |
|---|---|---|
| 1 | 74 | 80 |
| 3 | 72-80-187 | 72-81-187 |
| 5 | 70-72-74-81-187 | 70-72-74-81-187 |
| 10 | 47-69-70-72-74-79-80-187-311-312 | 47-69-70-72-74-78-80-103-187-312 |
| 15 | 47-51-69-70-72-74-78-80-86-88-91-92-96-187-312 | 47-51-69-70-72-74-78-80-86-88-91-93-96-187-312 |

| b | CR=150 | CR=200 |
|---|---|---|
| 1 | 81 | 80 |
| 3 | 72-80-187 | 72-80-187 |
| 5 | 70-72-74-80-187 | 70-72-74-81-187 |
| 10 | 47-68-70-72-74-78-80-103-187-312 | 47-68-70-72-74-78-80-103-187-312 |
| 15 | 47-51-68-70-72-74-78-80-86-88-91-93-187-311-312 | 47-51-68-70-72-74-78-80-86-88-91-92-187-311-313 |

Model (6) produces consistent results in terms of nodes interdicted for comparable $b$ levels even though the number of routes in the network ($R$) or cost ratios ($CR$) are different (i.e. see Tables 2 and 3). Even though solution times increase gradually as the interdictor's budget ($b$) and the number of routes ($R$) increase, our two stage model is solved to optimality within less than 6.3

seconds for the instance with $R = 1840$ and $b = 15$.

*4.3. Rerouting decisions after disruption(s)*

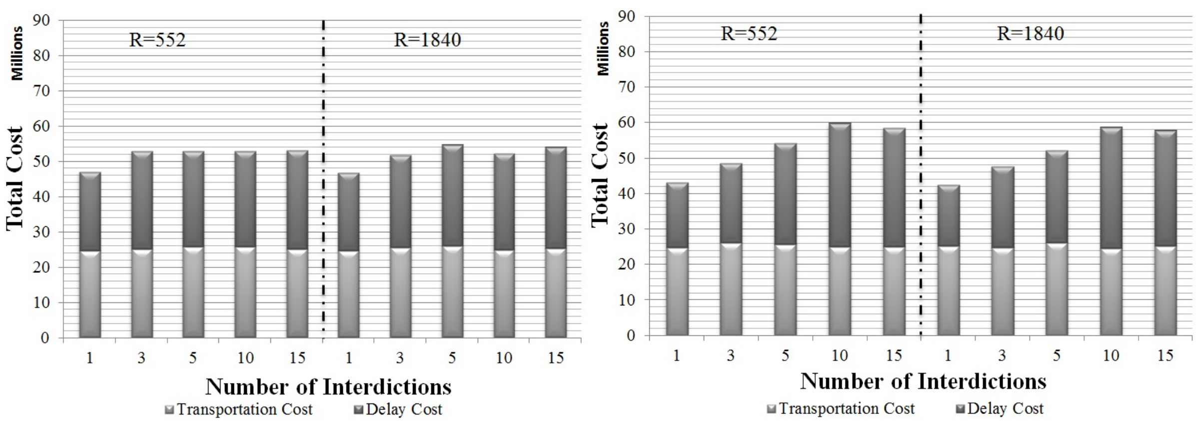
In the previous section, we demonstrated how total transportation and delay costs respond to varying $b$, $CR$, and $R$. In this section, we introduce a Google Maps-based tool that displays the routes along which the unit trains are moving ($X_{rt}$) and waiting ($O_{rt}$) at different time periods $\triangle$. As it can be seen in Figure 2, one can select the number of routes ($R$), number of interdictions allowed ($b$) and the specific time period ($\triangle$) as input parameters. Then, the solution of model (6) (i.e. the values of $X_{rt}$ and $O_{rt}$ decision variables) is displayed on a map.

Figure 2 shows mines, plants, and interdicted nodes, as well as the routes along which the trains are dispatched and delays occurred due to interdictions. In order to display the delays and movements of trains clearly, straight lines are used to draw the routes between mines and plants. However, in reality, those straight lines stand for shortest paths between selected origin and destination nodes. Moreover, the more frequently a route is being used, the thicker the red straight line becomes to represent the intensity of the route. Similarly, the more frequently delays occur on a route, the thicker the blue straight line is drawn to highlight the intensity of the delays on that route. The blue and red routes seen on Figure 2 demonstrate all train delays and dispatches over the planning period. On the other hand, a very detailed demonstration of the train movements and delays for different time periods can also be monitored by the decision tool as exemplified in Figure 3. Finally, gray routes in both Figures 2 and 3 represent the routes that are not being used or there is no delay is being incurred on at that time period.

*4.4. Impacts of capacity and demand levels*

In Section 4.2, it was observed that the impact of an attacker's budget ($b$) has a negligible impact on the total transportation cost. On the other hand, significant increases are observed in total delay cost as the budget increases in all scenarios even within the same $CR$ zones (see Figures 1a and 1b). Being able to measure the capacity of each arc and node enables us to assess the impacts of interdictions more precisely. Note that arc and node capacity constraints (3c) and (3d) are already able to assess the capacity of nodes and arcs in terms of unit trains for a given specific amount of time. For this case study, it is assumed that once a node is interdicted, then the capacity of incoming and outgoing arcs is also set to zero as well as the capacity of the node itself. On the other hand, in some situations, it is possible that the same interdiction might affect the capacity of other non-interdicted nodes and arcs as well (huge explosion, flood etc.). Such capacity adjustments can be made easily with the help of constraints (3c) and (3d). Moreover, the same capacity reduction technique can be employed when other trains use the same tracks and rail yards. In such circumstances, the capacity constraints should be adjusted so that the impacts of congestion can be reflected in the model.

In order to account for the scenarios mentioned above, Figure 4 illustrates the impacts of interdictions under different demand and node capacity levels for different network sizes when $CR = 100$.




(a) Increasing the demands of all plants by 100 %

(b) Increasing both the demands of all plants and node capacities by 100 % and 50%, respectively

Figure 4: Transportation and delay cost when $CR = 100$

Figures 4a and 4b illustrate the total transportation and delay costs with respect to the interdictor's budget level when demands of all plant nodes are increased by 100 %. In addition, all node capacities are raised by 50 % to reflect the changes in cost terms in Figure 4b. The effect of budget level on total transportation cost is at the minimum level in Figures 4a and 4b. However, increases in demand levels lead to a proportional increase in transportation costs since the number of unit trains needs to be transported between origin destination pairs is increased.

Due to the increase in demand levels, delay costs also increase due to the congestion at rail yards and tracks. Moreover, demand increases result in larger delay costs in such a way that the delay costs for larger and smaller budget levels are brought close together as evidenced in Figure 4a. In addition to the 100 % increase in demand levels, increasing all node capacities by 50 % reduces the delay costs for the cases when the interdictor's budget is small (i.e. 1, 3) as shown in Figure 4b.

However, an increase in delay cost is observed when the budget level is high ($\geq 5$). This is because increasing demands and/or node capacities enables an attacker to interdict different rail elements. Such differences can be seen in Tables A.1 and A.2.

## 5. Conclusions

This study aims to identify the impacts of vulnerable infrastructure elements in a real rail transportation network. We describe the problem elements and boundaries and discuss the commonly encountered model formulations in the literature where the vulnerability of the network is the point of interest. Then, we introduce a dynamic-discrete capacitated, time-variant consequence estimation model as opposed to a static-continuous uncapacitated, time-invariant model. The required reformulation to reduce the bi-level max-min problem into a single level max problem in case of disruptions to the real coal case transportation network is explained. Finally, we analyze the results of the computational experiments.

One of the main contributions of this paper to the literature is that the proposed mathematical model captures the movement of unit trains in time and space over a finite time horizon and identifies the critical nodes in the network whose unavailability causes the largest destruction in terms of total transportation and delay costs. The impacts of disruptions on objectives for different scenarios are demonstrated and rerouting decisions are illustrated via a web-based tool which provides useful insights for decision makers in planning further activities on the same rail transportation network. For each scenario, we are able to demonstrate the values of our decision variables on a map: (i) number of trains waiting for departure for route $r$ in time period $t$, ($O_{rt}$) (ii) number of trains moving on route $r$ in time period $t$, $X_{rt}$.

Our results point out that increasing the number of disruptions has almost no impact on transportation cost whereas, the delay cost due to congestions in rerouting unit trains dramatically escalates. This is because, the usage of the $K$-shortest path algorithm to construct the initial set of routes provides almost identical paths between each origin and destination pairs in terms of total distance. However, having multiple good alternative routes is not enough to prevent the delays of unit trains as the number of interdiction increases.

There are several directions that should be considered as next steps in modeling the impacts of disruptions on the rail network. First, we would like to incorporate the flow of unit trains in the reverse direction (plants to mines) as well while considering the routing decisions. The second direction is to incorporate the uncertainty associated with the occurrence of disruptions using a stochastic programming framework.

## Appendix A. Appendix

### *Appendix A.1. Trimming Algorithm*

The first phase of identifying critical elements in the rail network was handled by Algorithm Appendix A.1. All critical components of the network (i.e. tunnels, bridges) included in the $K$-shortest path algorithm are added to the reduced network that was later used to test the two stage interdiction model. Following notation introduced below includes some definitions of the terms used in trimming algorithm Appendix A.1.

- $Y$ =set of railyards
- $\mathcal{N}$ =set of nodes "Real Nodes"
- $B$ =set of elementary edges that have bridges (an elementary arc may or may connect two nodes)
- $\mathcal{Q}$ =set of edges that connect two nodes
- $n_{ij}^k$= set of elementary nodes on the $k$th shortest path between $i$ and $j$
- $a_{ij}^k$= set of elementary edges on the $k$th shortest path between $i$ and $j$
- $m_{ij}$ =the number of paths between $i$ and $j$
- $\delta_{ij}^k$ =the length of the $k$th shortest path between $i$ and $j$
- $d_{ij}$ = length of edge $(i, j)$

The first two steps connect each real node to its closest real node neighbor. In the first step, the algorithm checks if any two real nodes are connected without any other real node in between. If so, the edge connecting that pair of real node is added to the edge set $\mathbb{E}$ (step 1) and for each member of this set, a dummy path is created (step 2). In step 3, If there is no other real node found in between, then the original path is preserved with its original components (rail line distances, nodes etc.).

After step 2, the real nodes become connected to one another. However, actual distances and nodes in between real nodes are still unknown. In the rest of the steps, the algorithm generates the K-th shortest paths for each node pairs connected by arcs in edge set $\mathbb{E}$. During this stage, critical elements embedded in the paths $\in \mathbb{E}$ such as tunnels, bridges are also identified. Once a critical element is detected (i.e. a bridge with $m$ as beginning node and $n$ as end node) between node $i$ and $j$, edges $(i, m)$, $(m, n)$ and $(n, j)$ are created and added to $\mathcal{Q}$. Nodes defining the critical element ($m$ and $n$), are also added to $\mathcal{N}$ as well.

**Algorithm Appendix A.1** Procedure for constructing a connected network from USCA data

Set $\mathcal{N} = M \cup P \cup Y$

Let $\mathbb{E} = \emptyset$ be an empty set of edges.

1. Connect each node to its closest neighbors

    for each node pair $(i,j) \in \mathcal{N}$ such that $i \neq j$

        IF $n_{ij}^1$ does not contain a node in the set $\mathcal{N}$, THEN add $(i,j)$ to $\mathbb{E}$

2. Add dummy paths

    for each arc $(i,j) \in \mathbb{E}$

        add a dummy path from $i$ to $j$ that is composed of the single edge $(i,j)$
        set $a_{ij}^{m_{ij}+1} = (i,j)$
        set $\delta_{ij}^k$ to a large number

3. Add alternates for routes that have vulnerable elementary arcs

    for each arc $(i,j) \in \mathbb{E}$
      Set $k = 1$
      Set interdictionCost=0
      while $k \leq m_{ij}$+1
        for each $(\ell, p) \in a_{ij}^k \cap B$
          interdictionCost+=$f_{\ell m}$
        IF $a_{ij}^k \cap B \neq \emptyset$ and interdictionCost$\leq b$
          for each $(\ell, m) \in a_{ij}^k \cap B$
            add $(i,\ell)$, $(\ell,p)$, and $(p,j)$ to $\mathcal{Q}$
            add $\ell$ and $p$ to $\mathcal{N}$
            set $d_{i\ell} = \delta_{i\ell}^1$, $d_{\ell p} = \delta_{\ell p}^1$, and $d_{pj} = \delta_{pj}^1$
        ELSE
          add $(i,j)$ to $\mathcal{Q}$
          set $d_{ij} = \delta_{ij}^k$
          break from while loop
       $k = k + 1$

4. Compute capacity of arcs

    for each arc $(i,j) \in \mathcal{Q}$

RETURN the graph defined by nodes $\mathcal{N}$, edges $\mathcal{Q}$, and distances $(d_{ij})_{(i,j)}$

Table A.1: Interdicted nodes when $CR$=100 and demands of all plants are increased by 100 %

| b | R=552 | R=1840 |
|---|---|---|
| 1 | 80 | 81 |
| 3 | 70-72-80 | 70-72-74 |
| 5 | 70-72-74-80-187 | 70-72-74-80-187 |
| 10 | 70-71-72-73-74-75-80-81-186-187 | 63-68-70-72-74-78-80-103-187-312 |
| 15 | 68-69-70-71-72-73-74-75-78-79-80-81-102-186-187 | 47-68-69-70-71-72-73-74-75-78-79-80-81-180-186 |

Table A.2: Interdicted nodes when $CR$=100 and demands of all plants and node capacities are increased by 100 % and 50%, respectively

| b | R=552 | R=1840 |
|---|---|---|
| 1 | 73 | 73 |
| 3 | 70-72-74 | 72-80-187 |
| 5 | 70-72-74-80-187 | 70-72-74-81-187 |
| 10 | 69-70-72-74-78-80-86-88-103-187 | 47-68-70-72-74-78-80-103-187-312 |
| 15 | 51-69-70-72-74-78-80-83-84-86-88-91-93-102-187 | 47-51-68-70-72-74-78-80-86-88-91-92-96-187-312 |

## References


[1] Abril, M., Barber, F., Ingolotti, L., Salido, M., Tormos, P., and Lova, A. (2008). An assessment of railway capacity. *Transportation Research Part E: Logistics and Transportation Review*, 44(5):774–806.

[2] Ahuja, R. K., Cunha, C. B., and Sahin, G. (2005). *Network models in railroad planning and scheduling*, volume 1, pages 54–101.

[3] Assad, A. (1980). Models for rail transportation. *Transportation Research Part A: General*, 14(3):205–220.

[4] Babick, J. P. (2009). Tri-level optimization of critical infrastructure resilience. Master's thesis, Naval Postgraduate School.

[5] Bleizeffer, D. (2005). Record coal prices: another train derailment compounds export delays.

[6] Boyd, J. D. (2011). Dhs warns of al qaeda rail plot.

[7] Burdett, R. and Kozan, E. (2006). Techniques for absolute capacity determination in railways. *Transportation Research Part B: Methodological*, 40(8):616–632.

[8] Church, R. L., Scaparra, M. P., and Middleton, R. S. (2004). Identifying critical infrastructure: The median and covering facility interdiction problems. *Annals of the Association of American Geographers*, 94(3):491–502.

[9] Cormican, K., Morton, D. P., and Wood, R. K. (1998). Stochastic network interdiction. *Operations Research*, 46(2):184–197.

[10] Crainic, T. G. (2000). Service network design in freight transportation. *European Journal of Operational Research*, 122(2):272–288.

[11] Fulkerson, D. R. and Harding, G. C. (1977). Maximizing minimum Source-Sink path subject to a budget constraint. *Mathematical Programming*, 13(1):116–118.

[12] Haefner, L., Goodwin, R., and Porrello, L. (1996). The great flood of 1993: Impacts on waterborne commodity flow, rail transportation, and surrounding region. In *1996 Semisesquicentennial Transportation Conference Proceedings*.

[13] Israeli, E. and Wood, R. K. (2002). Shortest-Path network interdiction. *Networks*, 40(2):97–111.

[14] Janjarassuk, U. and Linderoth, J. (2008). Reformulation and sampling to solve a stochastic network interdiction problem. *Networks*, 52(3):120–32.

[15] Jespersen-Groth, J., Potthoff, D., Clausen, J., Huisman, D., Kroon, L., Maróti, G., and Nielsen, M. (2009). Disruption management in passenger railway transportation. In Ahuja, R., Möhring, R., and Zaroliagis, C., editors, *Robust and Online Large-Scale Optimization*, volume 5868 of *Lecture Notes in Computer Science*, chapter 18, pages 399–421. Springer Berlin / Heidelberg, Berlin, Heidelberg.

[16] Kohl, N., Larsen, A., Larsen, J., Ross, A., and Tiourine, S. (2007). Airline disruption management: Perspectives, experiences and outlook. *Journal of Air Transport Management*, 13(3):149–162.

[17] Kozan, E. and Burdett, R. (2005). A railway capacity determination model and rail access charging methodologies. *Transportation Planning and Technology*, 28(1):27–45.

[18] Lawley, M., Parmeshwaran, V., Richard, J., Turkcan, A., Dalal, M., and Ramcharan, D. (2008). A time-space scheduling model for optimizing recurring bulk railcar deliveries. *Transportation Research Part B: Methodological*, 42(5):438–454.

[19] Lim, C. and Smith, J. C. (2007). Algorithms for discrete and continuous multicommodity flow network interdiction problems. *IIE Transactions*, 39(1):15–26.

[20] Mattsson, L.-G. (2007). Railway capacity and train delay relationships. In Murray, A. T. and Grubesic, T. H., editors, *Critical Infrastructure*, Advances in Spatial Science, pages 129–150. Springer Berlin Heidelberg.

[21] Nemani, A. K. and Ahuja, R. K. (2011). *OR models in freight railroad industry.*

[22] Pan, F. and Morton, D. P. (2008). Minimizing a stochastic maximum-reliability path. *Networks*, 52(3):111–119.

[23] Peterson, S. K. and Church, R. L. (2008). A framework for modeling rail transport vulnerability. *Growth and Change*, 39(4):617–641.

[24] Qi, X., Bard, J. F., and Yu, G. (2004). Supply chain coordination with demand disruptions. *Omega*, 32(4):301–312.

[25] Qi, X., Bard, J. F., and Yu, G. (2006). Disruption management for machine scheduling: The case of SPT schedules. *International Journal of Production Economics*, 103(1):166–184.

[26] Quillen, E. (1997). Rail merger brings delays, derailments. *High Country News*, (118).

[27] Rocco, C. M., Ramirez-Marquez, J. E., and Salazar, D. E. (2010). Bi- and Tri-Objective optimization in the deterministic network interdiction problem. *Reliability Engineering and System Safety*, 95(8):887–896.

[28] Rocco, C. M. S., Salazar, D. E. A., and Ramirez-Marquez, J. E. (2009). Multi-objective network interdiction using evolutionary algorithms. Piscataway, NJ, USA.

[29] Royset, J. O. and Wood, R. K. (2007). Solving the bi-objective maximum-flow network-interdiction problem. *INFORMS Journal on Computing*, 19(2).

[30] Sherali, H. D. and Alameddine, A. (1992). A new reformulation-linearization technique for bilinear programming problems. *Journal of Global Optimization*, 2(4):379–410.

[31] U.S. Coal Activity Map. United states coal activity map. `http://www.platts.com/products/uscoalactivitymap`. Accessed:2013-08-02.

[32] U.S. Energy Information Administration (2010). Electric power annual 2008.

[33] U.S. Energy Information Administration (2012). Coal transportation rates to the electric power sector.

[34] Wollmer, R. (1964). Removing arcs from a network. *Operations Research*, 12(6):934–940.

[35] Xia, Y., Yang, m.-H., Golany, B., Gilbert, S. M., and Yu, G. (2004). Real-time disruption management in a two-stage production and inventory system. *IIE Transactions*, 36(2):111–125.

[36] Yen, J. Y. (1971). Finding the k shortest loopless paths in a network. *Management Science*, 17(11):pp. 712–716.

[37] Zhu, Z., Bard, J. F., and Yu, G. (2005). Disruption management for resource-constrained project scheduling. *Journal of the Operational Research Society*, 56(4):365–81.

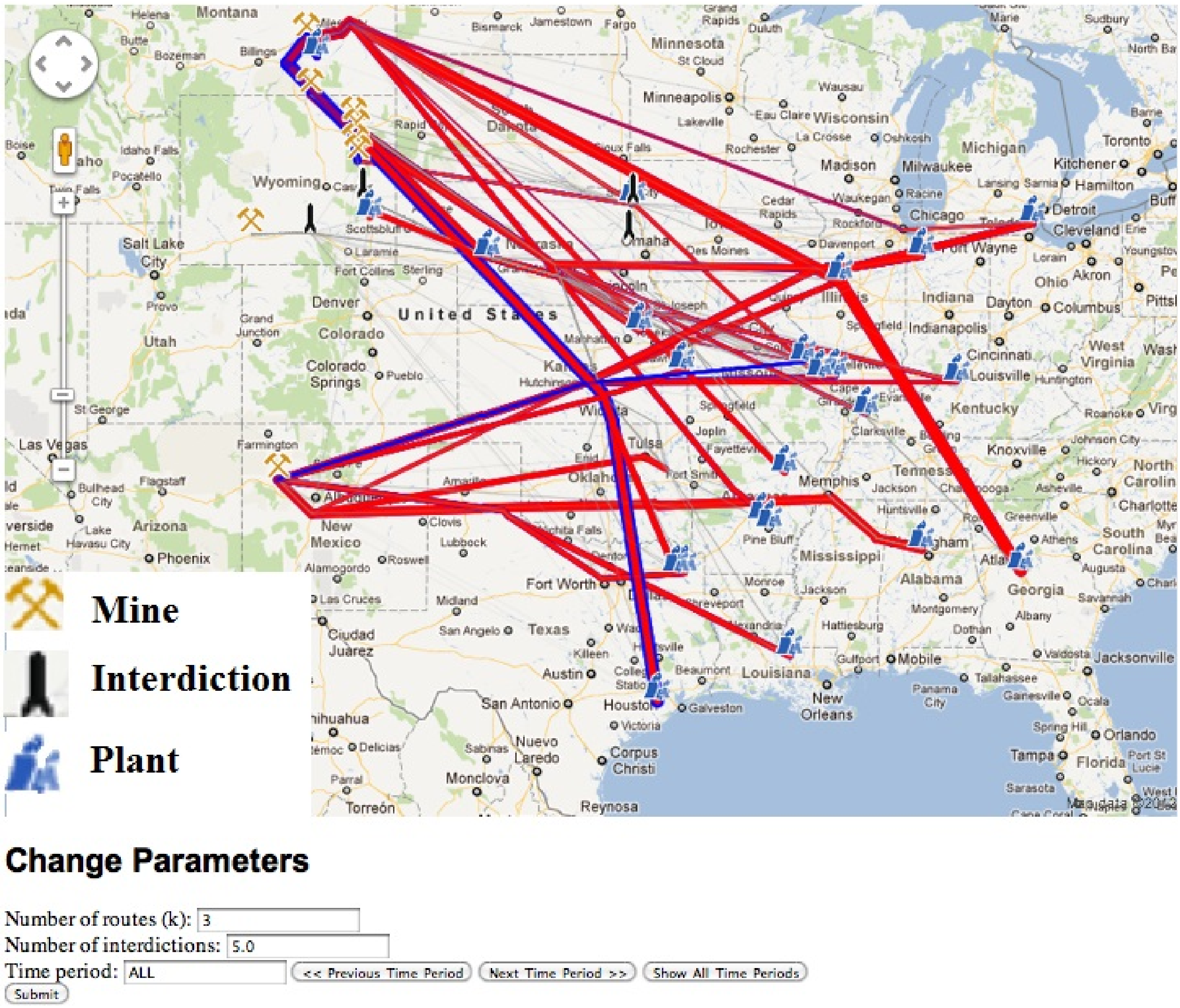
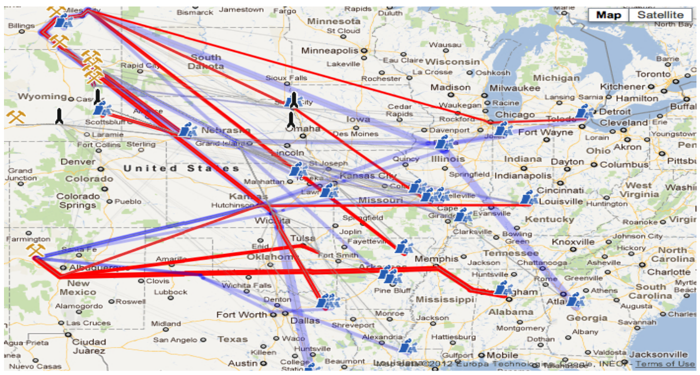
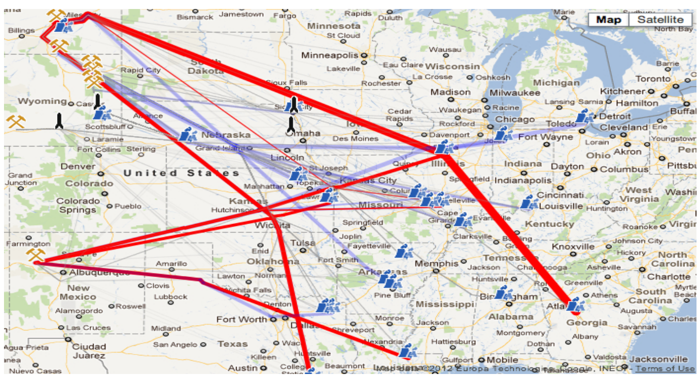



Figure 2: Web based network interdiction and resilience visualization tool



(a) $O_{rt}$ and $X_{rt}$ at $\triangle = 5$



(b) $O_{rt}$ and $X_{rt}$ at $\triangle = 10$

Figure 3: Number of trains waiting and/or moving on routes when $R = 552$, $b = 5$ and $CR$=100