Vulnerability Analysis of Initial Attack in Suppressing the Worst case Wildfire (Pyro-terrorism)

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Aims of this study

- Developing a mathematical programming model for the problem of vulnerability analysis of initial attack when responding to worst case wildfires and pyro-terrorism.

- Using the model to Assess the vulnerability of initial attack when responding to pyro-terrorism for a Test Case problem.

- Investigating the impact of Response Delay, Fire Growth Rate, Amount of Suppression Resources, Fire Ignition Locations, and Fire Station Locations on IA performance/success.

Initial Attack

- Initial Attack (IA) is defined as the first suppression action taken against wildfires to protect lives and property, and prevent further extension of the fire.

- In an IA, the resources that have been already allocated (deployed) to fire bases are dispatched to fires. Resources can be engines (small and large), crews, dozers and helicopters.

Problem Description

The problem is to identify the vulnerability of IA when responding to worst case wildfires and pyro-terrorism. For a given initial attack resource deployment, we investigate whether the IA can control a worst case wildfire, based on the location and number of fires.

Assumptions

- Adversaries have **complete knowledge** about the geographical characteristics of the landscape, the wind direction, rate of spread of fire and major fire spread direction in each cell.
- A wildfire spreads through the network using paths with the **minimum travel time (MTT)**.
- The duration for IA is T.
- Cell r is burnt when wildfire reaches its center.
- Fire spreads in an **elliptical shape** within each cell.
- The **fire line construction rates** are known for all fire suppression resources.
- Wildfire managers have **perfect knowledge** of the amount of resources required to control a fire.

Problem Description:

A Stackelberg Game Approach

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Having the knowledge about fire bases and fire suppression resources, attack the landscape with multiple ignition points fires to maximize the number of fires that do not receive sufficient resources to be contained (i.e. maximizing the number of escaped fires)

After observing the fire ignition locations set by the adversaries, optimally dispatch fire suppression resources from fire bases to fire locations to contain fires and minimize the number of fires that escape initial attack.

Modeling Fire Spread

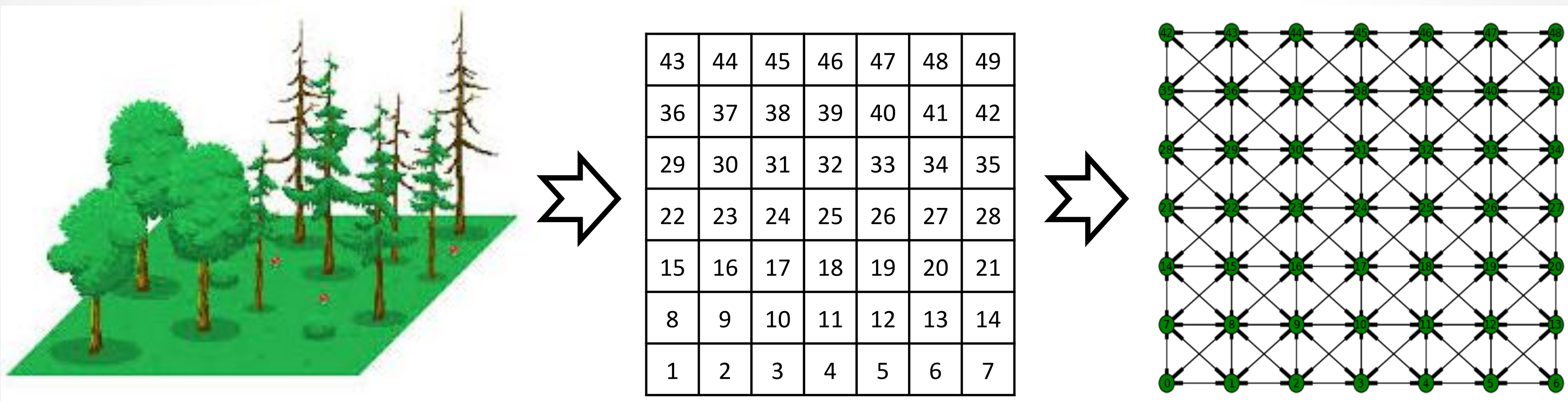

Fig. 1. Modeling a landscape to a grid network

Rate of Spread of Fire

$$R = \frac{b^2 - c^2}{b - c \times \cos(\theta)} \quad (0 \leq \theta < \pi/2) \qquad (1)$$

$$R = \frac{b^2 - c^2}{b + c \times \cos(\pi - \theta)} \quad (\pi/2 \leq \theta < \pi) \qquad (2)$$

- *b* and *c* are standard parameters describing an ellipse.
- *c* denotes half of the distance between two foci.
- *b* denotes half of the length of the major axis.

Notations:

- Sets and indices:

C is the set of raster cells in a landscape indexed with j;

F is the set of potential fire ignition locations indexed with f;

B is the set of operating bases indexed with b;

R is the set of resources type indexed with r;

- Variables:

Z_f 1 if the fire ignited at location f is contained with initial attack, 0 otherwise;

W_f 1 if a fire is ignited at location f, 0 otherwise (vector **W**);

$Y_{b,r,f}$ the number of resources of type r dispatched from operating base b to the fire ignited at location f;

Notations

- Parameters:

$\Phi_{b,r,f}$	fire line built within time frame T by resource r dispatched from operating base b to fire f;
π_f	total perimeter of the fire ignited at location f at the end of fire duration T;
$Q_{b,r}$	the number of resources of type r available at operating base b;
β	Budget: the maximum number of fires ignited;

Vulnerability Assessment of Initial Attack Problem (VAIAP)

$$\text{VAIAP: } A^* = \max_{\boldsymbol{W}\in\Xi}\left(\min_{(\boldsymbol{Y},\boldsymbol{W})\in\Psi(\boldsymbol{W})} \sum_{f\in F}(1 - Z_f)\right) \qquad (1)$$

Where the set Ξ is defined as the set of all $\boldsymbol{W}$ such that

$$\sum_{f\in F} W_f \leq \beta \qquad (2)$$

$$W_f \in \{0, 1\} \qquad \forall\ f \in F \qquad (3)$$

Vulnerability Assessment of Initial Attack Problem (VAIAP)

and the set $\Psi(\boldsymbol{W})$ is defined by

$$\pi_f Z_f W_f \leq \sum_{b\in B} \sum_{r\in R} \Phi_{b,r,f} Y_{b,r,f} \qquad \forall f \in F \qquad (4)$$

$$\sum_{f\in F} Y_{b,r,f} \leq Q_{b,r} \qquad \forall b \in B, \forall r \in R \qquad (5)$$

$$Y_{b,r,f} \in \mathbb{Z} \qquad \forall f \in F \quad \forall b \in B, \forall r \in R \qquad (6)$$

$$Z_f \in \{0,1\} \qquad \forall f \in F \qquad (7)$$

Fire Containment Condition

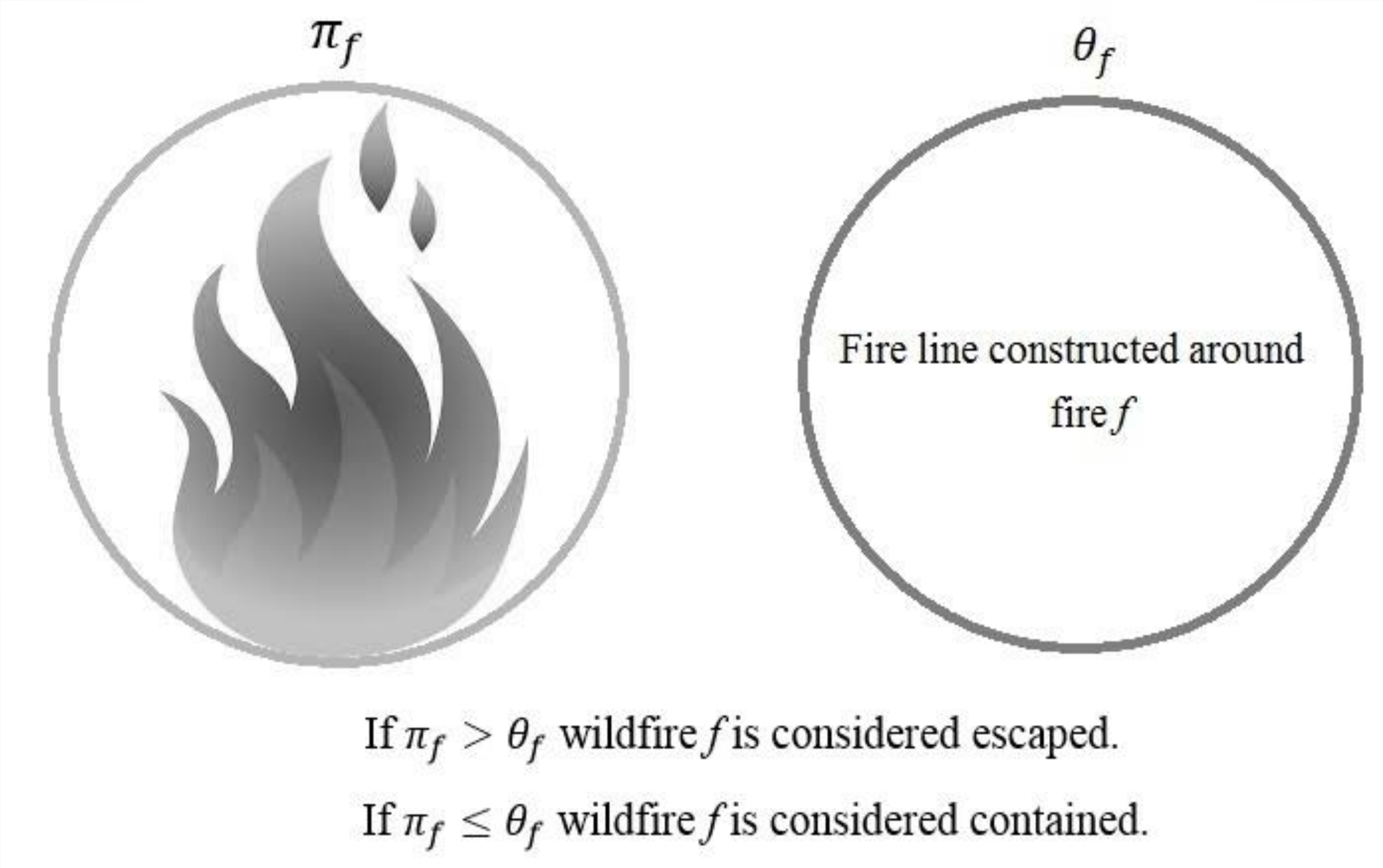

Fig. 2. Containment condition

Solution Methodology

Bounded Decomposition Algorithm (BDA)

The Dispatching Problem (DP) for a known pyro-terror attack

$$\mathrm{DP}(\widehat{\boldsymbol{W}}):\ A_{min}(\widehat{\boldsymbol{W}}) = \min \sum_{f \in \Delta} (1 - Z_f) \qquad (8)$$

$$\pi_f Z_f \widehat{W}_f \leq \sum_{b \in B} \sum_{r \in R} \Phi_{b,r,f} Y_{b,r,f} \qquad \forall f \in \Delta \qquad (9)$$

$$\sum_{f \in \Delta} Y_{b,r,f} \leq Q_{b,r} \qquad \forall b \in B,\ \forall r \in R \qquad (10)$$

$$Z_f \in \{0,1\},\ Y_{b,r,f} \in \mathbb{Z} \qquad \forall f \in \Delta,\ \forall b \in B, \forall r \in R \qquad (11,12)$$

Bounded Decomposition Algorithm (BDA)

$$\text{WCWP: } \Lambda = \max \sum_{c \in C} X_c \qquad (14)$$

$$X_c \leq \sum_{f \in F} H_{c,f} \times W_f \qquad \forall c \in C \qquad (15)$$

$$\sum_{f \in F} W_f \leq \beta \qquad (16)$$

$$W_f \in \{0, 1\} \qquad \forall f \in F \qquad (17)$$

$$X_c \in \{0, 1\} \qquad \forall c \in C \qquad (18)$$

Lower Bound on IA Capability

$$\text{LB: } \Theta_{LB} = min \sum_{b,r,f} \Phi_{b,r,f} Y_{b,r,f} \qquad (19)$$

$$\sum_{f \in F} W^{LB}{}_{f} = \beta \qquad (20)$$

$$\sum_{b \in B} \sum_{r \in R} Y_{b,r,f} \leq \sum_{b \in B} \sum_{r \in R} Q_{b,r} W^{LB}{}_{f} \qquad \forall f \in F \qquad (21)$$

$$\sum_{f \in F} Y_{b,r,f} = Q_{b,r} \qquad \forall\, b \in B, r \in R \qquad (22)$$

$$Y_{b,r,f} \in \mathbb{Z} \qquad \forall f \in F, b \in B, r \in R \qquad (23)$$

$$W^{LB}{}_{f} \in \{0, 1\} \qquad \forall f \in F \qquad (24)$$

Lower Bound on IA Capability

Theorem 1:

Θ_{LB} is a lower bound on the capability of IA in containing any fire scenario when all the resources are dispatched.

Proof:

Any dispatching plan $\widehat{D}$ that uses all the resources to construct fire line around a set of fires is a feasible solution for LB, and, therefore would construct a fire line no less than Θ_{LB}. Lets' assume $\widetilde{D}(\boldsymbol{W})$ is the optimal dispatching plan for fire scenario $\boldsymbol{W}$ that would construct the fire line $\tilde{\Theta}(\boldsymbol{W})$ around an arbitrary fire scenario $\boldsymbol{W}$. Since $\widetilde{D}(\boldsymbol{W})$ is a feasible solution for LB, therefore $\Theta_{LB} \leq \tilde{\Theta}(\boldsymbol{W})$. ■

Steps of BDA

- **Step (1)** Set the iteration counter $k = 1$;
- **Step (2)** Solve the master problem WCWP and compute the fire perimeter $\Pi(\widehat{\boldsymbol{W}^k})$ for the optimal wildfire attack $\widehat{\boldsymbol{W}^k}$;
- **Step (3)** Solve the sub problem $DP(\widehat{\boldsymbol{W}^k})$ for the optimal wildfire attack $\widehat{\boldsymbol{W}^k}$.
- **Step (4)** If $A_{min}\left(\widehat{\boldsymbol{W}^k}\right) > 0$, stop; the resulting fire cannot be contained and IA is vulnerable to fire scenario $\widehat{\boldsymbol{W}^k}$.

- **Step (4-LB)** If $k = 1$ then solve the lower bound (LB) problem for dispatching model and identify the corresponding wildfire scenario for which these bounds hold. We refer to the set of corresponding fire location scenario for which LB is held as Γ ($\Gamma = \left\{f \in F \middle| \widehat{W^{LB}}_f = 1\right\}$). This step is only run once.

- **Step (5)** If the total perimeter of wildfire $\widehat{\boldsymbol{W}^k}$ is less than Θ_{LB}, then check whether $max\left\{\pi_f \middle| f \in \Delta\right\} < min\left\{\theta_f \mid f \in \Delta\right\}$, if so stop; there is no wildfire attack that cannot be contained;
- **Step (6)** If $A_{min}\left(\widehat{\boldsymbol{W}^k}\right) = 0$, then set $k = k + 1$, add constraint (25) to the master problem and go to Step (3).

Test Case Problem – Santa Fe National Forest

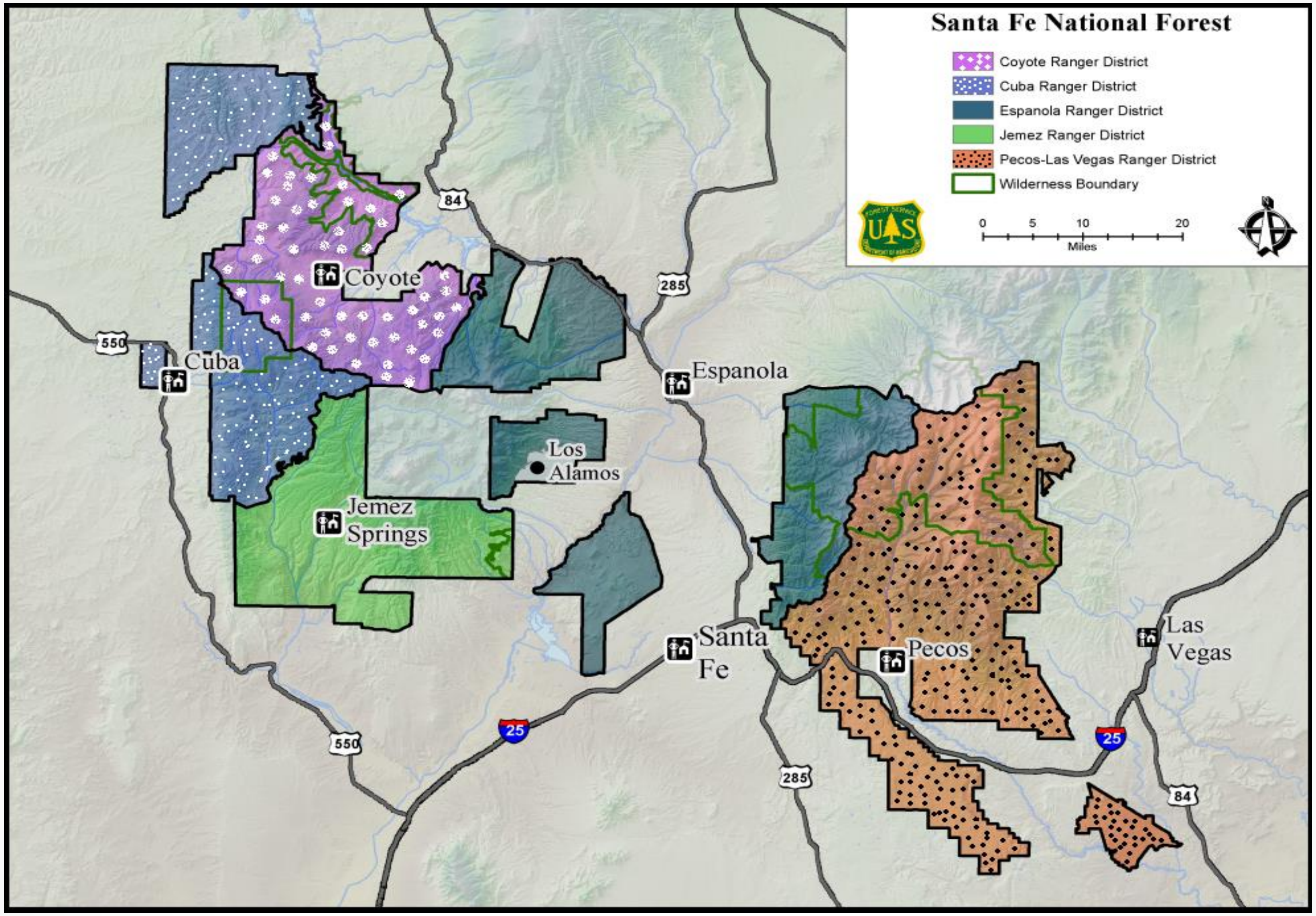

Fig. 3. Ranger districts, Santa Fe National Forest.

Initial Attack Resources

Table 1. The average hourly line production rate of a 3-person crew small engine, and a 4-person crew large engine

Resource	Average fire line production rate (m/h)
Small Engine (type 6)	272
Large Engine (type 3 and 4)	380

Table 2. Fire stations and their approximate distances to the landscape test case

Fire station	Distance from the landscape (km)
Cuba	8.96
Coyote	22.72
Jemez	14.27
Espanola	60.87
Pecos	95.82

Results

Table 3. The computational results for an IA with 30 (*min*) response time limit for a worst-case wildfire with 30 (*min*) response delay

Response Time (minute)	Num. ignition points	Total fire perimeter	Fire line constructed	Num. fires escaped
30	1	795	174	1
	2	1,504	174	2
	3	2,244	171	3

IA Vulnerability

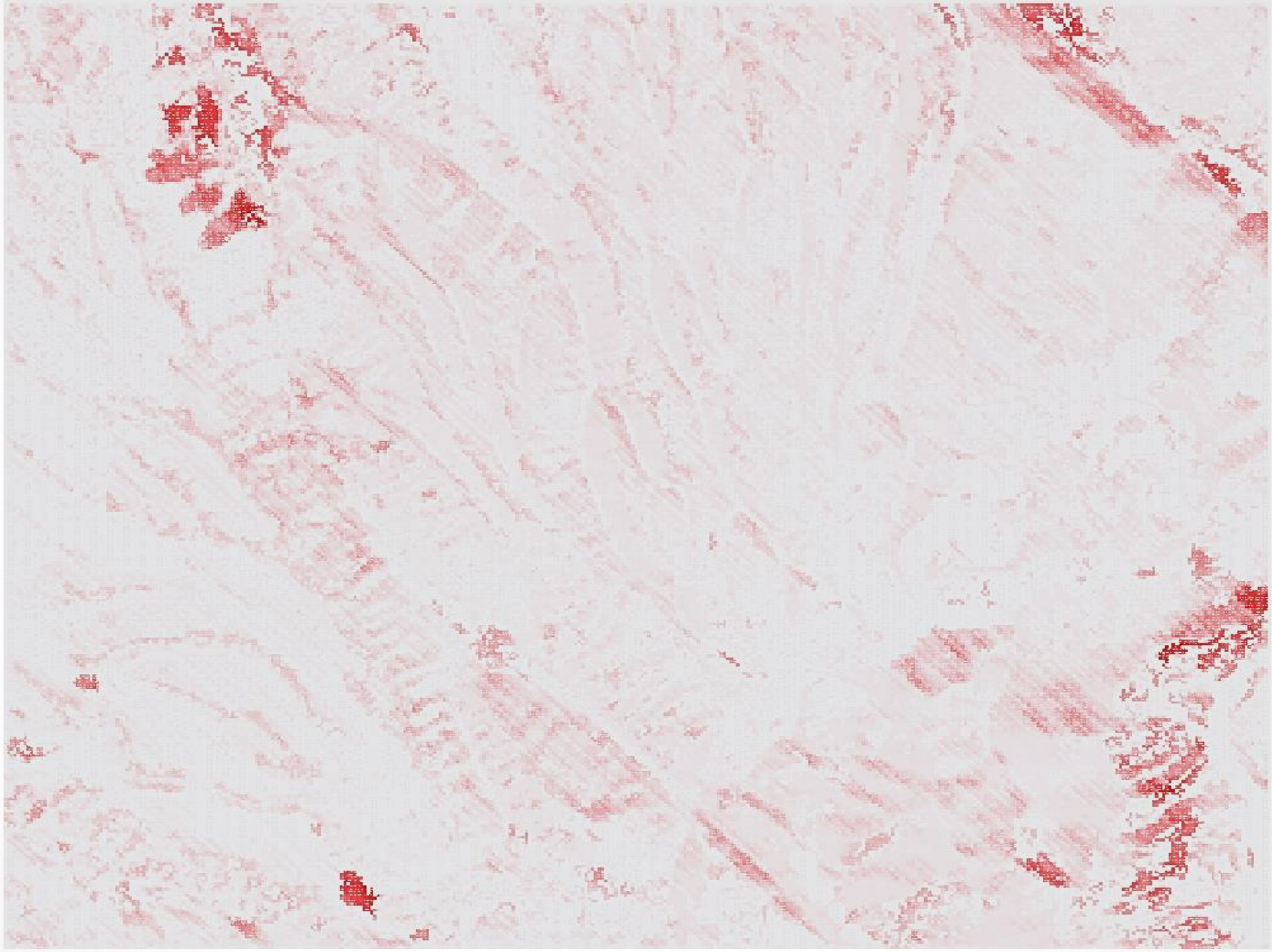

Fig. 4. The heat map of the IA-vulnerability for fires with single ignition point. The darker a cell color is, the more IA-vulnerable that cell is

Results

Table 4. The computational results for an IA with 60 (*min*) response time limit for a worst-case wildfire with 30 (*min*) response delay

Response Time (minute)	Num. ignition points	Total fire perimeter	Fire line constructed	Num. fires escaped
60	1	927	975	0
	2	1,848	976	1
	3	2,759	976	2

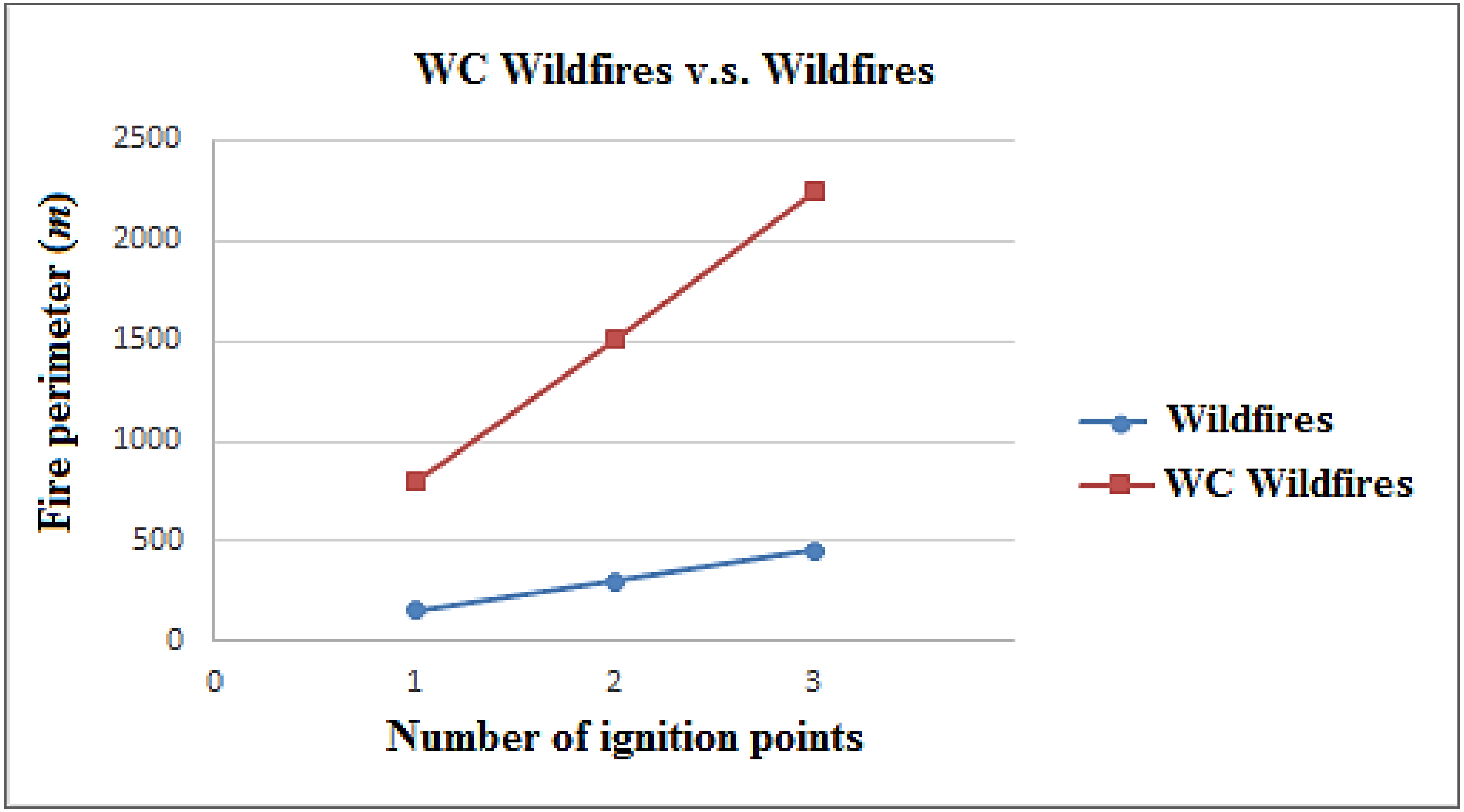

Fig. 5. Fire perimeters for worst-case wildfires (WC Wildfires) with carefully selected ignition points, and the average fire perimeters after 30 (*min*) for (natural) wildfires with randomly located ignition points (for a sample with 5000 ignition location scenarios)

Table 5. The computational results for an IA with 30/60 (*min*) response time limit for natural wildfires (with randomly located ignition points) with 30 (*min*) response delay

Response Time (minute)	Num. ignition points	Ave. total fire perimeter	Ave. fire line constructed	% of scenarios in which at least a fire escapes IA
30	1	151	185	25
	2	303	185	86
	3	453	204	100
60	1	216	1,023	0
	2	413	1,024	1
	3	643	1,038	17

Sensitivity Analysis

Response Delay

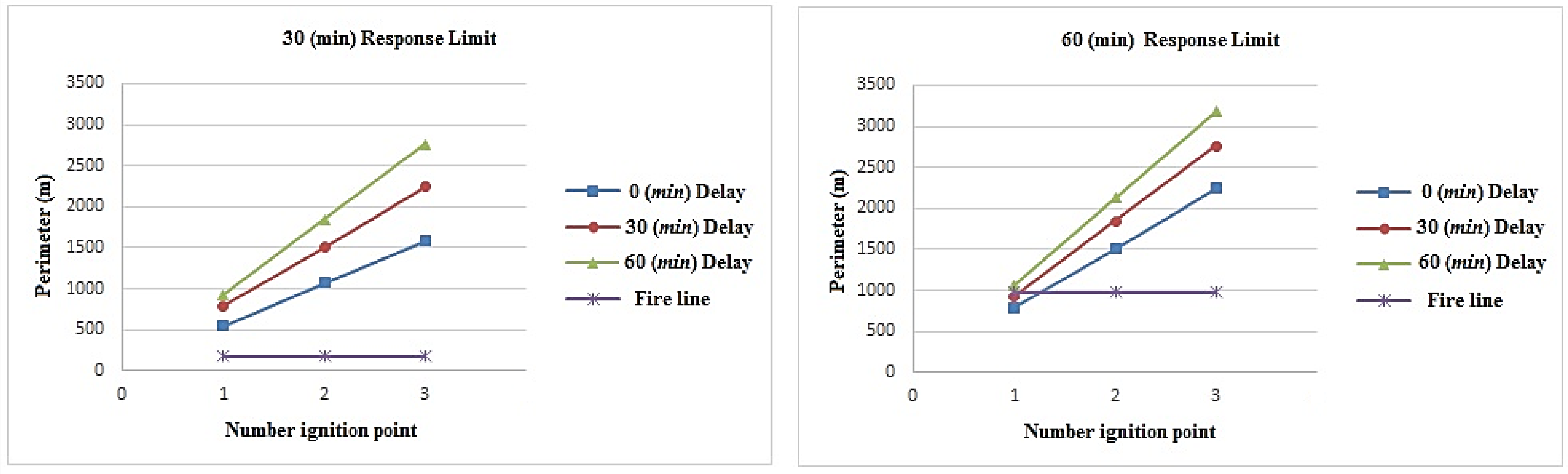

Fig. 6. Fire perimeters for different response delays

Sensitivity Analysis

Fire Growth Rate

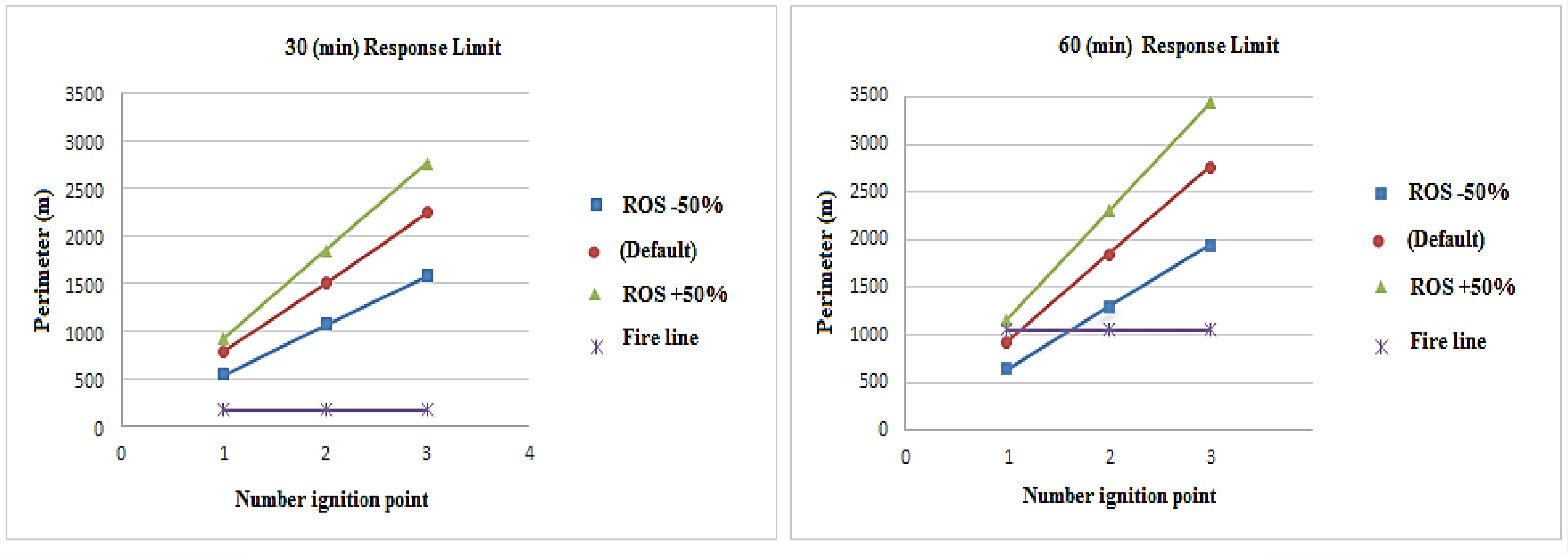

Fig. 7. Fire perimeter for fires with different ROS

Sensitivity Analysis

Fire Growth Rate

Fig. 8. The heat map of the average ROS in each cell of the landscape. The darker a cell color is, the higher the ROS in that cell is.

Summary and Results

- This study presents an optimization model and algorithm to the problem of vulnerability analysis of Initial Attack when responding to pyro-terrorism and worst case wildfires.

- The problem is modeled as a Stackelberg game problem in which the adversaries, as the leader, knowing the location and the amount of initial attack resources, start a multiple ignition points wildfire in a landscape, and the fire managers, as the follower, respond to the wildfire by an optimal dispatching of resources to fires.

- A decomposition algorithm is developed to solve the problem. The algorithm computes a lower bound on the initial attack capability and uses that as a stopping criteria.

Summary and Results

- A test case landscape based on Santa Fe National Forest is used for experimentation. The results suggest that ROS of wildfires has a great impact on the IA success in containing that fire, such that the heat map of rate of spread of wildfires is merely the same as that of IA-vulnerability. Therefore, conducting fuels management in the landscape can increase the chances of IA success in containing pyro-terrorism and worst case wildfires

- Improving wildfire detection can enhance IA success in containing wildfires, before they grow large and become difficult to suppress

Question?